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SIMILARITY AND DECAY LAWS OF MOMENTUMLESS WAKES.(U)
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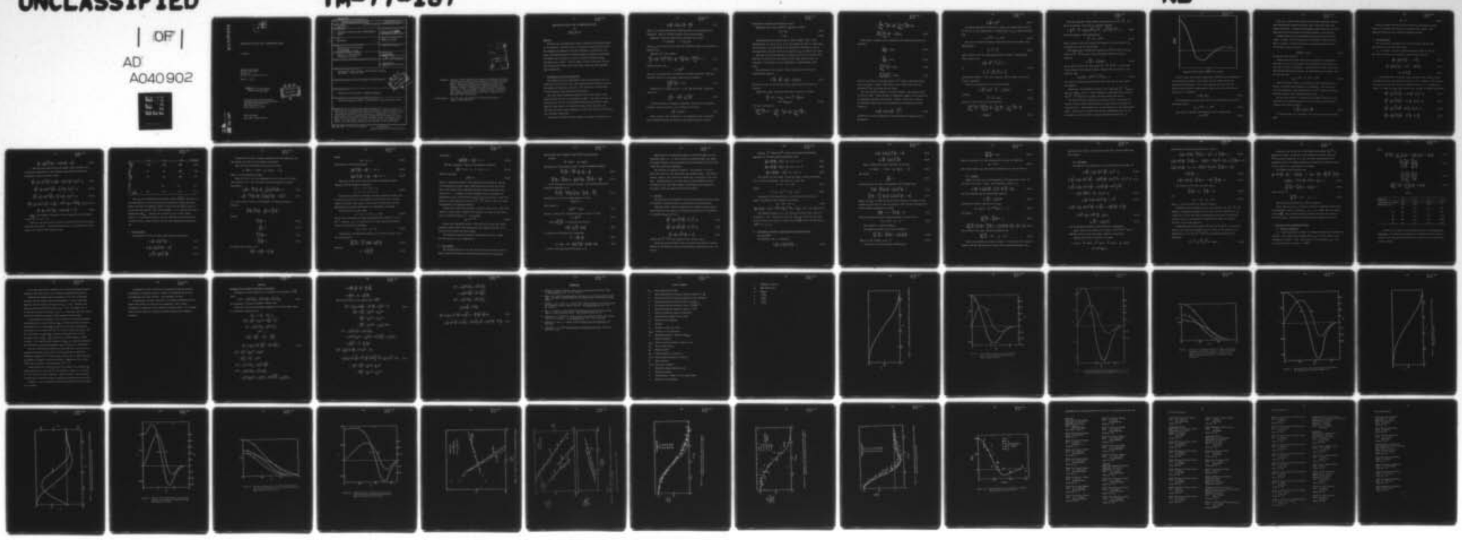

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S. Hassid

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SIMILARITY AND DECAY LAWS OF MOMENTUMLESS WAKES

by

Samuel Hassid

Abstract

The decay laws of momentumless wakes as predicted by different models are investigated. Existing work is critically reviewed, and the models are compared with themselves and with experiments. Subsequently, the decay of momentumless wakes is examined using the Reynolds stress models of Launder, Reece and Rodi, as well as a simpler turbulent energy - dissipation (k-ϵ) model. The last model is shown to be better than the more sophisticated former ones, and exhibits generally fair agreement with existing data

1. *Introduction and Literature Review*

The similarity laws of momentumless wakes have attracted a lot of attention in the last ten years, both because of their practical interest and because of theoretical interest, since turbulence in these wakes is much different than turbulence in drag wakes.

The first fully documented experiment on the wake behind a self-propelled body was the one of Naudascher (1965). The main conclusion of this author is that in a self-propelled wake production decays faster than the other terms in the turbulent energy balance equation, and therefore this flow may be regarded as a small mean velocity perturbation on a turbulent energy wake.

The relevant turbulent energy equation, according to Naudascher, is:

$$U_o \frac{\partial k}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r}(\nu_T r \frac{\partial k}{\partial r}) - \frac{10\nu k}{\lambda^2} \tag{1.1}$$

where ν_T is an eddy diffusivity coefficient and λ is the microscale of turbulence. Both are assumed constant in each cross-section.

Naudascher looked for self similar solutions of the kind:

$$k = K(x)h(\eta) \qquad \eta = r/r_{1/2}(x)$$

where $r_{1/2}$ is the value of r at which the turbulent energy is one quarter is maximum value.

Equation (1.1) thus becomes:

$$\frac{d^2h}{d\eta^2} + \frac{1}{\eta}\frac{dh}{d\eta} + \frac{r_{1/2}U_o}{\nu_T}\frac{dr_{1/2}}{dx}\eta\frac{dh}{d\eta} - \frac{U_o r_{1/2}^2}{\nu_T K}\frac{dK}{dx}h - \frac{10\nu K r_{1/2}^2}{\lambda^2 \nu_T}h = 0 \tag{1.2}$$

Assuming further that:

$$\nu_T = L_m K^{1/2} \tag{1.3}$$

where L_m is the macroscale of turbulence, suitably normalized Naudascher concludes that, for a self-similar solution to be obtained

$$\frac{r_{1/2}U_o}{L_m K^{1/2}}\frac{dr_{1/2}}{dx} = \text{Const.} \tag{1.4}$$

Furthermore, by integrating Eq. 1.2 over the flow field Naudascher shows that

$$\frac{10\nu}{U_o\lambda^2} = -\frac{1}{K}\frac{dK}{dx} - \frac{2}{r_{1/2}}\frac{dr_{1/2}}{dx} \tag{1.5}$$

To obtain a power law solution, Naudascher assumes that Loitsiansky's parameter remains constant at the centerline of the flow:

$$KL_m^5 = \text{Constant} \tag{1.6}$$

Since, however, this assumption is not confirmed by his experimental data, Naudascher divides the flow into three decay regions, in which

Loitsiansky's parameter has different values.

Naudascher also tried a different approach, in which:

$$L_m \sim r_{1/2} \tag{1.7}$$

$$\lambda \sim L_m \tag{1.8}$$

This approach gives no power law; it rather suggests that λ tends asymptotically to a value of λ_∞. It is not thought that this is a physically sound behaviour, and experiments do not confirm it (although, of course, the experiments show that the rate of growth of $r_{1/2}$ is indeed very small).

The main flaw of both approaches of Naudascher is due to the fact that both Eqs. (1.6) and (1.8) are relevant to the final period of decay of turbulence, and therefore are not appropriate to high Reynolds number flows.

For the decay of the velocity defect, Naudascher starts from the axial momentum equation:

$$U_o \frac{\partial U_d}{\partial x} + \frac{\partial \overline{u^2}}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{uv}) \tag{1.9}$$

Notice that in Eq. (1.9) the axial stress and pressure terms were not neglected.

Naudascher, again, looked for self-similar solutions in which:

$$\frac{U_d}{U_D} = f(\eta), \quad P/P_{min} = p(\eta), \quad \overline{u^2} = \overline{u^2_{max}}h(\eta)$$

$$\overline{uv} = \overline{uv}_{max}g(\eta) \tag{1.10}$$

Thus Eq. (1.9) becomes:

$$\frac{U_o r_{1/2}}{\overline{uv}_{max}}\frac{dU_D}{dx} f(\eta) - \frac{U_o U_D}{\overline{uv}_{max}} \frac{dr_{1/2}}{dx} \eta \frac{df}{d\eta} + \frac{r_{1/2}}{\overline{uv}_{max}} \frac{d\overline{u^2_{max}}}{dx} h$$

$$- \frac{\overline{u^2}_{max}}{\overline{uv}_{max}} \frac{dr_{1/2}}{dx} \eta \frac{dh}{d\eta} + \frac{r_{1/2}}{\overline{uv}_{max}} \frac{1}{\rho} \frac{dP_{min}}{dx} p(\eta)$$

$$- \frac{dr_{1/2}}{\rho dx} \frac{P_{min}}{\overline{uv}_{max}} \eta \frac{dp}{d\eta} = - \frac{1}{\eta} \frac{d}{d\eta}(\eta g) \tag{1.11}$$

Self-similar solutions may be obtained if the following relations are satisfied:

$$\frac{P_{min}}{\rho U_o U_D} = \text{Const.} \tag{1.12}$$

$$\frac{\overline{u^2}_{max}}{U_o U_D} = \text{Const.} \tag{1.13}$$

$$\frac{U_o U_D}{\overline{uv}_{max}} \frac{dr_{1/2}}{dx} = \text{Const.} \tag{1.14}$$

$$\frac{U_D \, dr_{1/2}/dx}{r_{1/2} \, dU_D/dx} = \text{Const.} \tag{1.15}$$

The main conclusion is that $U_o U_D$ decays as $\overline{u^2}$, which is supported fairly well by his data. However, his analysis is of little value since his predicted $\overline{u^2}$ and $r_{1/2}$ decays are not right.

The next important analysis of the wake behind a self-propelled body is that of Finson. Finson assumes, as Naudascher suggests, that production is negligible compared to dissipation. However, Finson uses a form of the turbulent energy equation more appropriate to high Reynolds number turbulence.

$$U_o \frac{\partial k}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r}[\nu_T r \frac{\partial k}{\partial r}] - \frac{C_D k^{3/2}}{\Lambda} \tag{1.16}$$

In addition, he uses an equation for the characteristic length scale of turbulence Λ:

$$U_o \frac{d\Lambda}{dx} = C_k k^{1/2} \tag{1.17}$$

By chosing appropriate values for C_D and C_k, by integrating (1.16) and (1.17) over y, and assuming that Λ is proportional to $r_{1/2}$, Finson concludes that:

$$k \sim x^{-16/11} \qquad \Lambda \sim x^{3/11} \tag{1.18}$$

Now, for the momentum equation, Finson makes the usual boundary layer approximation:

$$\frac{P}{\rho} + \overline{v^2} = \frac{P_\infty}{\rho} \tag{1.19}$$

(Note, however, that this approximation does not apply to axisymmetric flows. In this case:

$$\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\partial \overline{v^2}}{\partial r} + \frac{\overline{v^2} - \overline{w^2}}{r} = 0$$

or (1.20)

$$\frac{P}{\rho} + \overline{v^2} + \int \frac{\overline{v^2} - \overline{w^2}}{r} dr = \frac{P_\infty}{\rho}$$

(see Townsend (1976)). This error, however, does not affect the rest of Finson's arguments).

With this assumption, Eq. (1.9) takes the form:

$$U_o \frac{\partial U_d}{\partial x} + \frac{\partial}{\partial x}(\overline{u^2} - \overline{v^2}) = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{uv}) \tag{1.21}$$

Assuming:

$$\overline{u^2} - \overline{v^2} = W\zeta(\eta) \tag{1.22}$$

Equation (1.21) obtains the following form:

$$\frac{r_{1/2}U_o}{\overline{uv}_{max}} \frac{dU_D}{dx} f - \frac{U_o U_D dr_{1/2}}{\overline{uv}_{max} dx} \eta \frac{df}{d\eta} + \frac{r_{1/2}}{\overline{uv}_{max}} \frac{dW}{dx} \zeta - \frac{W}{\overline{uv}_{max}} \frac{dr_{1/2}}{dx} \eta \frac{d\zeta}{d\eta}$$

$$= -\frac{1}{\eta}\frac{1}{d\eta}(g\eta) \tag{1.23}$$

From the equations, Finson deduces that U_D decays as $(\overline{u^2} - \overline{v^2})$. For this last quantity, Finson uses a transport equation:

$$U \frac{\partial}{\partial x}(\overline{u^2} - \overline{v^2}) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_T \frac{\partial(\overline{u^2} - \overline{v^2})}{\partial r}\right) - C_p \frac{k^{1/2}(\overline{u^2} - \overline{v^2})}{\Lambda} = 0 \qquad (1.24)$$

From this equation, Finson deduces that:

$$\overline{u^2} - \overline{v^2} \propto x^{-18/11} \qquad (1.25)$$

Notice, however, that this value of the exponent is too sensitive to C_p, a quantity far from well established!

Both Naudascher's argument that U_D decays as $\overline{u^2}$ and Finson's that U_D decays as $\overline{u^2} - \overline{v^2}$ are wrong if the self-similar solution for U_D in equation:

$$U_o \frac{\partial U_D}{\partial x} = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{uv}) \qquad (1.26)$$

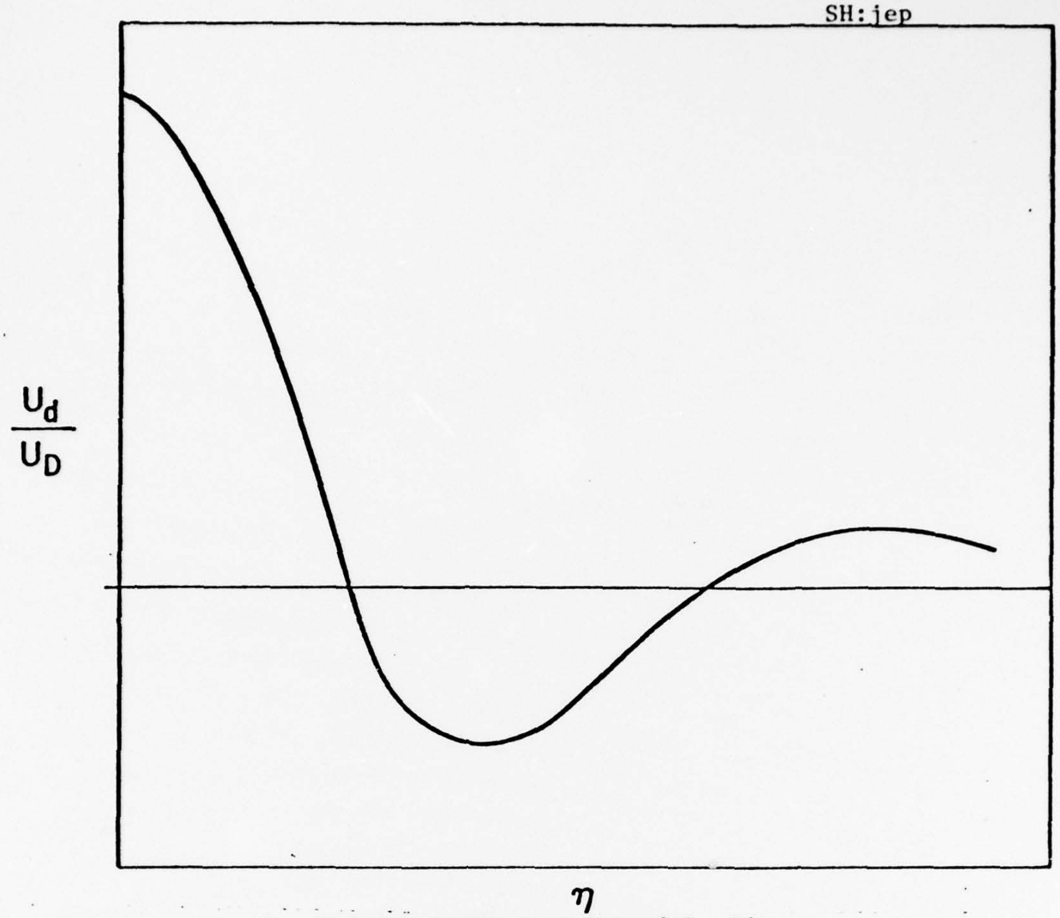
can be shown to decay more slowly than $(\overline{u^2} - \overline{v^2})$ (or $\overline{u^2}$). In that case, if one considers the momentum integral equations for the wake of a self-propelled body:

$$U_o \int_0^\infty U_D r\,dr + \int_0^\infty \left(\overline{u^2} + \frac{P - P_\infty}{\rho}\right) r\,dr = 0 \qquad (1.27)$$

the second term decays faster than the first term, and the wake becomes momentumless.

Furthermore, in Naudascher's results, it is shown that $(\overline{u^2} - \overline{v^2})_{max}$ is equal to $.03U_oU_D$, and it is rather difficult to understand how a quantity like $\overline{u^2} - \overline{v^2}$ would drive a quantity 30 times larger than itself.

The rather arbitrary equation for the decay of U_D is the reason why Finson's velocity defect profile shows the slight positive overshoot at high values of η. This overshoot has not been observed in real momentumless wakes. (Of course, one should state that the experimental error in the measurement of axial velocity is rather high for high values of η.)



Velocity defect profile predicted by Finson.

At this stage, one can give a theoretical justification why production can be expected to decay to zero faster than the other terms in the turbulent energy balance. For, if the opposite were true, then all velocity scales would be proportional to each other (as is the case for drag wakes, for example).

$$K \propto U_D^2 \propto \overline{uv}_{max} \tag{1.28}$$

The self-similar solution would then be the one predicted by Tennekes and Lumley (1973)

$$U_D \sim x^{-4/5}, \quad k \sim x^{-8/5} \tag{1.29}$$

Now, Finson's analysis and Naudascher results indicate that:

$$K \sim x^{-16/11} \tag{1.30}$$

Thus, one is lead to the absurd conclusion that turbulence behind a self-propelled body would decay faster than a turbulent energy wake without production. Therefore the assumption that production is of the same order as the other terms in the energy balance is unacceptable.

Another analysis of the momentumless wake was given by Lin & Pao (1974). However, in this analysis the wake behind a propeller driven body is examined, in which conservation of angular momentum leads to the following integral equation:

$$\int_0^\infty W r^2 dr = \text{Const.} \tag{1.31}$$

W being the swirl velocity.

Though production by the axial velocity gradient is negligible in this wake, it is not at all obvious that production by the gradient of the swirl velocity is negligible. Assuming that the swirl velocity and the turbulent fluctuating velocity are proportional to each other, Lin & Pao conclude that:

$$r_{1/2} \sim x^{1/4} \qquad W \sim K^{1/2} \propto x^{-3/4} \tag{1.32}$$

Notice that these values are very close to the ones predicted by Finson, who assumed that production is negligible. Also notice that Lin & Pao themselves, after deducing that $W \sim K^{1/2}$, neglect production when they calculate the turbulent energy profile.

For the axial velocity, Lin and Pao assume that $\overline{u^2}$ and P/ρ can be neglected in the momentum equation, and use an eddy viscosity model to calculate the velocity profile:

$$U_o \frac{\partial U_d}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}[r\nu_T \frac{\partial U_d}{\partial r}] \tag{1.33}$$

By assuming that ν_T is constant in the cross-section, they conclude that:

$$U_D \sim x^{-1} \qquad (1.34)$$

Since all models outlined in this section have some minor or major flaws, it is proposed in this work to study the wake behind a self-propelled body using three different turbulence models.

2. The Models Used

The self-similar momentumless low velocity defect wake has been calculated using the following models.

a. A k-ε model, in which a gradient diffusion model is used for $\overline{uv}$, whereas k and ε are described by the following equations.

$$\frac{Dk}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{6}\frac{k^2}{\varepsilon}\frac{\partial k}{\partial y}\right] - \varepsilon - \overline{uv}\,\frac{\partial U}{\partial y} \qquad (2.1)$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{12}\frac{k^2}{\varepsilon}\frac{\partial \varepsilon}{\partial y}\right] - 1.9\frac{\varepsilon^2}{k} - 1.44\frac{\varepsilon}{k}\overline{uv}\,\frac{\partial U}{\partial y} \qquad (2.2)$$

$$-\,\overline{uv} = \frac{1}{6}\frac{k^2}{\varepsilon}\frac{\partial U}{\partial y} \qquad (2.3)$$

b. The simple model of Launder, Reece and Rodi (1975) (Model LI) with a modified value for the diffusion constant in the ε equation. This constant was modified so that k^2/ε tends to a constant value as $y \to \infty$. Without this change, the eddy viscosity could tend to zero, and this is highly undersirable.

$$\frac{D\overline{u^2}}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{4}\frac{\overline{v^2}k}{\varepsilon}\frac{\partial \overline{u^2}}{\partial y}\right] - 1.2\overline{uv}\,\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\overline{u^2} + \frac{\varepsilon}{3} \qquad (2.4)$$

$$\frac{D\overline{v^2}}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{4}\frac{\overline{v^2}k}{\varepsilon}\frac{\partial \overline{v^2}}{\partial y}\right] - .4\overline{uv}\,\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\overline{v^2} + \frac{\varepsilon}{3} \qquad (2.5)$$

$$\frac{D\overline{w^2}}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{4}\frac{\overline{v^2}k}{\varepsilon}\frac{\partial \overline{w^2}}{\partial y}\right] - .4\overline{uv}\,\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\overline{w^2} + \frac{\varepsilon}{3} \qquad (2.6)$$

$$\frac{D\overline{uv}}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{4}\frac{\overline{v^2}k}{\varepsilon}\frac{\partial \overline{uv}}{\partial y}\right] - .4\overline{v^2}\,\frac{\partial U}{\partial y} - \frac{3}{2}\varepsilon\frac{\overline{uv}}{k} \qquad (2.7)$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial y}\left[\frac{1}{8}\frac{\overline{v^2}k}{\varepsilon}\frac{\partial\varepsilon}{\partial y}\right] - 1.44\frac{\varepsilon}{k}\,\overline{uv}\,\frac{\partial U}{\partial y} - 1.9\frac{\varepsilon^2}{k} \tag{2.8}$$

c. The more sophisticated model of Launder, Reece and Rodi (LII), in which the rapid pressure redistribution terms and the three-point-correlation are given invariant forms:

$$\frac{D\overline{u^2}}{Dt} = \frac{\partial}{\partial y}\left[.11\frac{k}{\varepsilon}\left(\overline{v^2}\frac{\partial\overline{u^2}}{\partial y} + 2\overline{uv}\,\frac{\partial\overline{uv}}{\partial y}\right)\right] - \frac{11.6}{11}\,\overline{uv}\,\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\,\overline{u^2} + \frac{\varepsilon}{3} \tag{2.9}$$

$$\frac{D\overline{v^2}}{Dt} = \frac{\partial}{\partial y}\left[.11\frac{k}{\varepsilon}\left(3\overline{v^2}\,\frac{\partial\overline{v^2}}{\partial y}\right)\right] - \frac{4}{11}\,\overline{uv}\,\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\,\overline{v^2} + \frac{\varepsilon}{3} \tag{2.10}$$

$$\frac{D\overline{w^2}}{Dt} = \frac{\partial}{\partial y}\left[.11\frac{k}{\varepsilon}\left(\overline{v^2}\,\frac{\partial\overline{w^2}}{\partial y}\right)\right] - \frac{6.4}{11}\,\overline{uv}\,\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\,\overline{w^2} + \frac{\varepsilon}{3} \tag{2.11}$$

$$\frac{D\overline{uv}}{Dt} = \frac{\partial}{\partial y}\left[.11\frac{k}{\varepsilon}\left(2\overline{v^2}\,\frac{\partial\overline{uv}}{\partial y} + \overline{uv}\,\frac{\partial\overline{v^2}}{\partial y}\right)\right] - \frac{(2.6\overline{v^2} + 2k - 1.2\overline{u^2})}{11}\frac{\partial U}{\partial y} - \frac{3}{2}\frac{\varepsilon}{k}\,\overline{uv} \tag{2.12}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial y}\left[.165\frac{k}{\varepsilon}\,\overline{v^2}\,\frac{\partial\varepsilon}{\partial y}\right] - 1.44\frac{\varepsilon}{k}\,\overline{uv}\,\frac{\partial U}{\partial y} - 1.9\,\frac{\varepsilon^2}{k} \tag{2.13}$$

(Again, the diffusion coefficient in the ε equation has been changed so that $\frac{\overline{v^2}k}{\varepsilon} \to c$ as $y \to \infty$).

Initially, the self-similar solutions for plane wakes were investigated, using the three models. The following parameters, as computed by the three models, are compared with Townsend's data:

	k-ε	LI	LII	Townsend
$\frac{\overline{uv}_{max}}{U_D^2}$	-.047	-.039	-.035	-.061
$\frac{k_o}{U_D^2}$	.11	.11	.084	.105
$\frac{k_{max}}{U_D^2}$	.13	.12	.104	.12
$\frac{\overline{u_o^2}}{\overline{v_o^2}}$	-	1.6	1.3	.67
$K \equiv \frac{L}{x\theta^{1/2}}$	.23	.21	.19	.25

Here, U_D is the maximum velocity defect, whereas $\overline{u_o^2}/\overline{v_o^2}$ is the ratio of the longitudinal to cross-stream velocity fluctuations at the centerline. Notice that none of the models predicts a good value for $\overline{u_o^2}/\overline{v_o^2}$, since all of them indicate that $\overline{u_o^2} > \overline{v_o^2}$ at the centerline. It is seen that all models underestimate $\overline{uv}_{max}$ and the rate of spread K, but in this respect (which is, to the author's opinion, the most important) the gradient model is better than LI, which is better than its more sophisticated counterpart LII.

3. Gradient Model

The equations for the low defect momentumless plane wake are:

$$U_o \frac{\partial k}{\partial x} = \frac{\partial}{\partial y}\left[\frac{1}{6} \frac{k^2}{\varepsilon} \frac{\partial k}{\partial y}\right] - \varepsilon \tag{3.1}$$

$$U_o \frac{\partial \varepsilon}{\partial x} = \frac{\partial}{\partial y}\left[\frac{1}{12} \frac{k^2}{\varepsilon} \frac{\partial \varepsilon}{\partial y}\right] - 1.9\frac{\varepsilon^2}{k} \tag{3.2}$$

$$U_o \frac{\partial U_d}{\partial x} = \frac{\partial}{\partial y}\left[\frac{1}{6} \frac{k^2}{\varepsilon} \frac{\partial U_d}{\partial y}\right] \tag{3.3}$$

In both the k and the ε equation production has been neglected, and this neglect will have to be justified a posteriori.

Now, one seeks self-similar solutions of the type:

$$k = Kh(\eta) \quad \varepsilon = Ee(\eta) \quad U_d = U_D f(\eta) \quad \eta = \frac{y}{L}$$

where L is a characteristic length.

Since (3.1) and (3.2) are independent of (3.3), they will be dealt with separately (i.e., U_d is for most intents and purposes a passive contaminant.)

$$U_o \frac{dK}{dx} h - \frac{KU_o}{L} \frac{dL}{dx} \eta \frac{dh}{d\eta} = \frac{K^3}{EL^2} \frac{d}{d\eta}\left[\frac{1}{6} \frac{h^2}{e} \frac{dh}{d\eta}\right] - Ee \tag{3.4}$$

$$U_o \frac{dE}{dx} e - \frac{EU_o}{L} \frac{dL}{dx} \eta \frac{de}{d\eta} = \frac{K^2}{L^2} \frac{d}{d\eta}\left[\frac{1}{12} \frac{h^2}{e} \frac{de}{d\eta}\right] - 1.9\frac{e^2}{h} \frac{E^2}{K} \tag{3.5}$$

For a self-similar solution to be obtained, the following should be constants:

$$\frac{U_o}{E} \frac{dK}{dx}, \frac{K}{E} \frac{U_o}{L} \frac{dL}{dx}, \quad \frac{K^3}{E^2 L^2} \text{ and } \frac{KU_o}{E^2} \frac{dE}{dx}$$

setting:

$$-\frac{U_o}{E} \frac{dK}{dx} = \alpha \tag{3.6}$$

$$\frac{K^3}{E^2 L^2} = 1 \tag{3.7}$$

$$\frac{K}{E} \frac{U_o}{L} \frac{dL}{dx} \equiv \gamma \tag{3.8}$$

$$-\frac{U_o K}{E^2} \frac{dE}{dx} \equiv \beta \tag{3.9}$$

and noting that, from Eq. 3.7:

$$\frac{3}{E} \frac{dK}{dx} = 2 \frac{K}{E^2} \frac{dE}{dx} + 2 \frac{K}{E} \frac{1}{L} \frac{dL}{dx}$$

giving:

$$-3\alpha = -2\beta + 2\gamma \tag{3.10}$$

one obtains the following equations:

$$\frac{d}{d\eta}\left[\frac{1}{6}\,\frac{h^2}{e}\,\frac{dh}{d\eta}\right] + \gamma\eta\frac{dh}{d\eta} - e + \alpha h = 0 \tag{3.11}$$

$$\frac{d}{d\eta}\left[\frac{1}{12}\,\frac{h^2}{e}\,\frac{de}{d\eta}\right] + \gamma\eta\,\frac{de}{d\eta} - 1.9\frac{e^2}{h} + \beta e = 0 \tag{3.12}$$

Equations (3.10), (3.11) and (3.12) form an eigenvalue problem. (Together with the boundary conditions):

$$\begin{aligned} e = 1\ ,\quad h = 1 \quad \text{at } \eta = 0 \\ e \to 0\ ,\quad h \to 0 \quad \text{at } \eta \to \infty \end{aligned} \tag{3.13}$$

Because this problem is non-linear, one has to guess an initial eddy viscosity distribution to solve equation (3.11) and (3.12) by standard eigenvalue methods, then compute, from h and e, the new eddy viscosity distribution, and repeat the process till convergence is obtained.

The values of α, β, and γ are:

$$\alpha = 1.15 \quad \beta = 2.01 \quad \gamma = .292 \tag{3.14}$$

Notice that if one looks for power law solution of the type $k\propto^{n}$, $L\propto x^{n}$, $E\propto x^{n}$, where $n_\varepsilon = n_k - 1$, the values of α, β and γ give the following values for the power law exponents.

$$n_k = -1.33 \quad n_L = .33 \tag{3.15}$$

Turning now to the momentum equation, it is easy to see that it has the following form:

$$\frac{U_o K}{EU_D}\,\frac{dU_D}{dx}\,f - \frac{U_o K}{LE}\,\frac{dL}{dx}\eta\frac{df}{d\eta} = \frac{d}{d\eta}\left[\frac{h^2}{6e}\,\frac{df}{d\eta}\right] \tag{3.16}$$

Defining

$$\delta \equiv -\frac{K}{E}\,\frac{U_o}{U_D}\,\frac{dU_D}{dx} \tag{3.17}$$

one obtains

$$\frac{d}{d\eta}\left[\frac{h^2}{6e}\frac{df}{d\eta}\right] + \gamma\eta\frac{df}{d\eta} + \delta f = 0 \qquad (3.18)$$

The above equation, subject to the boundary conditions:

$$\frac{df}{d\eta} = 0 \text{ at } \eta = 0 \quad f \to 0 \text{ at } \eta \to \infty \qquad (3.19)$$

and the constraint:

$$\int_0^\infty f d\eta = 0 \qquad (3.20)$$

Form an eigenvalue problem. This problem has many solutions, but one is interested only in the lowest eigenvalue which satisfies Eq. (3.20). This is the second lowest eigenvalue, the first lowest eigenvalue being obviously $\delta = \gamma$. Although, strictly speaking, higher eigenvalues are momentumless wakes, the solutions present no physical interest, since they decay at a rate faster than the one of $(\overline{u^2} - \overline{v^2})$, which in this case cannot be neglected in the momentum equation.

Notice that if h^2/e were constant, δ would be equal to 3γ. Since h^2/e tends to a value which is somehow less than half its value at the center-line, one actually obtains $\delta = 3.5\gamma$, giving:

$$U_D \propto x^{-1.15} \qquad (3.21)$$

Notice that examination of Eqs. (3.21) and (3.15) shows that indeed production decays faster than dissipation, and confirms that Eqs. (3.1) and (3.2) are relevant for this problem.

The turbulent energy and velocity distribution for the k-ε model are shown in Fig. 1 and 2 respectively

4. The LI Model

The equations for k and ε are the same as the ones relevant to the k-ε model, because the turbulent energy becomes asymptotically isotropic.

One therefore has to consider only the $\overline{uv}$ and U_d equations.

Letting

$$\overline{uv} = Tg(\eta) \ , \quad U_d = U_D f(\eta)$$

one obtains, by substituting in Eqs. 2.7 and the momentum equation:

$$\frac{LU_o}{T}\frac{dU_D}{dx} f - \frac{U_o U_D}{T}\frac{dL}{dx}\eta\frac{df}{d\eta} = -\frac{dg}{d\eta} \tag{4.1}$$

$$\frac{U_o K}{TE}\frac{dT}{dx} g - \frac{KU_o}{EL}\frac{dL}{dx}\eta g' = \frac{d}{d\eta}[\frac{1}{6}\frac{h^2}{e}\frac{dg}{d\eta}] - \frac{K^2 U_D}{TEL}\frac{4h}{15} f' - \frac{3}{2}g\frac{e}{h} \tag{4.2}$$

If the solution is to be self-similar, the following expressions should be independent of x:

$$\frac{LU_o}{T}\frac{dU_D}{dx} , \quad \frac{U_o U_D}{T}\frac{dL}{dx} , \frac{U_o K}{TE}\frac{dT}{dx} , \quad \frac{KU_o}{EL}\frac{dL}{dx} \quad , \quad \frac{K^2 U_D}{TEL} \tag{4.3}$$

Remembering that one has already chosen L so that:

$$K^3/E^2L^2 = 1$$

this amounts to:

$$T/U_D K^{1/2} = \text{Const.} \tag{4.4}$$

Actually, without loss of generality one can chose T so that

$$T = U_D K^{1/2} \tag{4.5}$$

Letting $\frac{U_o K}{EU_D}\frac{dU_D}{dx} \equiv \delta$, and using the relation:

$$\frac{1}{T}\frac{dT}{dx} = \frac{1}{U_D}\frac{dU_D}{dx} + \frac{1}{2K}\frac{dK}{dx} \tag{4.6}$$

one obtains the following set of equations:

$$-\delta f - \gamma\eta\frac{df}{d\eta} = -\frac{dg}{d\eta} \tag{4.7}$$

$$-(\delta + \frac{\alpha}{2})g - \gamma\eta g' = \frac{d}{d\eta}[\frac{1}{6}\frac{h^2}{e}\frac{dg}{d\eta}] - \frac{4}{15}h\frac{df}{d\eta} - \frac{3}{2}\frac{e}{h}g \tag{4.8}$$

(' denotes differentiation with respect to η)

Again, this is an eigenvalue problem, to be solved subject to the constraint $\int_0^\infty f d\eta = 0$. It can be solved by standard methods; one should be careful, however, to use one matrix for both equations (3.7) and (3.8), rather than solve them separately.

The solution is pictured in figure 3. The value of $\delta = 3.3\gamma$ is nearly the same as in the solution for the gradient model. The velocity profile, however, has a kink in the middle and is too peaky in the outer side. Although there are no actual data for flat momentumless wakes, existing data for round wakes suggest a much higher ratio between the too peaks, and do not exhibit the kink in the middle.

5. LII Model

The behaviour of the Reynolds stress equation predicted by this model is remarkably different from the one predicted by the other models.

Starting, again, from the assumption that production decays faster than dissipation, one obtains the following equations for the components of turbulent energy, and for ε:

$$\frac{D\overline{u^2}}{Dt} = \frac{\partial}{\partial y}\left[.11\frac{\overline{v^2}k}{\varepsilon}\frac{\partial\overline{u^2}}{\partial y}\right] - \frac{3}{2}\varepsilon\frac{\overline{u^2}}{k} + \frac{\varepsilon}{3} \tag{5.1}$$

$$\frac{D\overline{v^2}}{Dt} = \frac{\partial}{\partial y}\left[.33\frac{\overline{v^2}k}{\varepsilon}\frac{\partial\overline{v^2}}{\partial y}\right] - \frac{3}{2}\varepsilon\frac{\overline{v^2}}{k} + \frac{\varepsilon}{3} \tag{5.2}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial y}\left[.165\frac{\overline{v^2}k}{\varepsilon}\frac{\partial k}{\partial y}\right] - 1.9\frac{\varepsilon^2}{k} \tag{5.3}$$

(Notice that $\overline{w^2} = \overline{u^2}$ in the asymptotic self-similar state.)

Unlike the previous models, this one predicts no return to isotropy, because of the different values of the diffusion coefficients in Eqs. 5.1 and 5.2.

Letting: $\overline{u^2} = K\phi(\eta)$ and $\overline{v^2} = K\chi(\eta)$, one obtains the following equations for the self-similar momentumless wake:

$$\frac{d}{d\eta}[.33\frac{\chi h}{e}\frac{d\chi}{d\eta}] - \frac{3}{2}\frac{e}{h}\chi + \frac{e}{3} + \gamma\eta\chi' + \alpha\chi = 0 \tag{5.4}$$

$$\frac{d}{d\eta}[.11\frac{\chi h}{e}\frac{d\phi}{d\eta}] - \frac{3}{2}\frac{e}{h}\phi + \frac{e}{3} + \gamma\eta\phi' + \alpha\phi = 0 \tag{3.5}$$

$$\frac{d}{d\eta}[.165\frac{\chi h}{e}\frac{de}{d\eta}] - 1.9\frac{e^2}{h} + \beta e + \gamma\eta e' = 0 \tag{5.6}$$

The solution of this system is shown in Figure 4 where the profiles of χ, ϕ and h are depicted. The values of γ and α are:

$$\gamma = .27 \qquad \alpha = +1.275 \tag{5.7}$$

giving

$$K \sim (x - x_o)^{-1.4} \qquad L \sim (x - x_o)^{.3} \tag{5.8}$$

Proceeding to find equations for the self-similar mean velocity profile and Reynolds stress profile, one obtains:

$$-g' + \gamma\eta f' + \delta f = 0 \tag{5.9}$$

$$\frac{d}{d\eta}[.11\frac{h}{e}(2\chi g' + g\chi')] - \frac{(2.6\chi + 2h - 1.2\phi)}{11}f' - \frac{3}{2}\frac{e}{h}g + \gamma\eta g' + (\delta + \frac{\alpha}{2})g = 0 \tag{5.10}$$

The predicted value of δ is .92, which is consistent with a value of n_u equal to -1.02. This value is too low, since n_u is more or less equal to n_k; the low value comes as a result of the $\overline{uv}\frac{\partial\overline{v^2}}{\partial y}$ term in the $\overline{uv}$ equation; without it, $n_u \simeq n_k$.

6. Self-Similar Solutions of Axisymmetric Momentumless Wakes

6.1 k-ε model

The equations for k, ε and U_d are:

$$U_o\frac{\partial k}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}[r\frac{1}{6}\frac{k^2}{\varepsilon}\frac{\partial k}{\partial r}] - \varepsilon \tag{6.1}$$

$$U_o \frac{\partial\varepsilon}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}[r\frac{1}{12}\frac{k^2}{\varepsilon}\frac{\partial\varepsilon}{\partial r}] - 1.9\frac{\varepsilon^2}{k} \tag{6.2}$$

$$U_o \frac{\partial U_d}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}[\frac{r}{6}\frac{k^2}{\varepsilon}\frac{\partial U_d}{\partial r}] \tag{6.3}$$

Again, seeking self-similar solutions of the type:

$$k = Kh(\eta) \quad \varepsilon = Ee(\eta) \quad U_d = U_D f(\eta) \quad \eta = \frac{r}{L} \tag{6.4}$$

and letting

$$\frac{K^3}{E^2L^2} = 1 \tag{6.5}$$

one obtains, for the first two equations, the following forms:

$$\frac{U_o}{E}\frac{dK}{dx}h - \frac{KU_o}{EL}\frac{dL}{dx}\eta\frac{dh}{d\eta} = \frac{1}{\eta}\frac{d}{d\eta}[\frac{\eta}{6}\frac{h^2}{e}\frac{dh}{d\eta}] - e \tag{6.6}$$

$$\frac{U_oK}{E^2}\frac{dE}{dx}e - \frac{KU_o}{E^2}\frac{dE}{dx}\eta\frac{de}{d\eta} = \frac{1}{\eta}\frac{d}{d\eta}[\frac{\eta}{12}\frac{h^2}{e}\frac{de}{d\eta}] - 1.9\frac{e^2}{h} \tag{6.7}$$

Again, this pair of equations can be solved (subject, of course, to the boundary conditions (3.13) and the relation 3.10), and the following value of the integral parameters are found:

$$\frac{U_o}{E}\frac{dK}{dx} = -1.2 \qquad \frac{U_o}{E}\frac{K}{L}\frac{dL}{dx} = .222 \tag{6.8}$$

which are consistent with the following power law coefficients:

$$n_k = -1.46 \quad n_L = .27 \tag{6.9}$$

The h profile is shown in Figure 6.

The equation relevant to the velocity defect is:

$$\frac{U_oK}{EU_D}\frac{dU_D}{dx}f - \frac{U_oK}{LE}\frac{dL}{dx}\eta f' = \frac{1}{\eta}\frac{d}{d\eta}(\eta\frac{h^2}{6e}\frac{df}{d\eta}) \tag{6.10}$$

subject to the condition $\int f\eta d\eta = 0$ (6.11)

The solution of this eigenvalue problem gives:

$$\frac{U_o K}{EU_D}\frac{dU_D}{dx} = -1.04 \tag{6.12}$$

which is consistent with the following form of power law behaviour:

$$n_u = -1.27 \tag{6.13}$$

The velocity defect and shear stress distribution are shown in Figure 7.

6.2 LI Model

Again, as in the case of plane geometry, the equation for k and ε are the same as for the k-ε model: the $\overline{uv}$ equation, however, is:

$$U_o \frac{\partial\overline{uv}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k^2}{6\varepsilon} r \frac{\partial\overline{uv}}{\partial r}\right] - \frac{k^2}{6\varepsilon}\frac{\overline{uv}}{r^2} - \frac{4k}{15}\frac{\partial U_d}{\partial r} - \frac{3}{2}\frac{\varepsilon}{k}\overline{uv} \tag{6.14}$$

with the following form of the momentum equation:

$$U_o \frac{\partial U_d}{\partial x} = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{uv}) \tag{6.15}$$

Seeking self-similar solutions of the type:

$$U_d = U_D f(\eta) \qquad \overline{uv} = K^{1/2} U_D g(\eta) \qquad r = L\eta \tag{6.16}$$

one obtains

$$\frac{U_o K}{EU_D}\frac{dU_D}{dx} f - \frac{U_o K}{EL}\frac{dL}{dx}\eta f' = g' \tag{6.17}$$

$$\left[\frac{U_o K}{EU_D}\frac{dU_D}{dx} + \frac{1}{2}\frac{U_o}{E}\frac{dK}{dx}\right]g - \frac{U_o K}{EL}\frac{dL}{dx}\eta g' = \frac{1}{\eta}\frac{d}{d\eta}\left[\eta\frac{h^2}{6e}\frac{dg}{d\eta}\right] - \frac{h^2}{6e}\frac{g}{\eta^2} - \frac{4h}{15}f' - \frac{3}{2}\frac{e}{h}g \tag{6.18}$$

The solution of the above eigenvalue problem gives:

$$\frac{U_o K}{EU_D}\frac{dU_D}{dx} = -.96\,, \qquad n_u = -1.17 \tag{6.19}$$

The f and g profiles are shown in Figure 8. Notice that this solution suffers from the same defects as the one of the plane momentumless wake:

the kink in the middle, the low peak to peak ratio, and the rather slow rate of decay .

6.3 LIII Model

The equations of axisymmetric flow consistent with this model are:

$$U_o \frac{\partial \overline{u^2}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[.11\frac{k}{\varepsilon} r\overline{v^2}\frac{\partial \overline{u^2}}{\partial r}\right] - \frac{3}{2}\frac{\varepsilon}{k}\overline{u^2} + \frac{\varepsilon}{3} \tag{6.20}$$

$$U_o \frac{\partial\overline{v^2}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[.33\frac{k}{\varepsilon}r\overline{v^2}\frac{\partial\overline{v^2}}{\partial r}\right] - .11\frac{k}{\varepsilon}\left[\frac{2\overline{vv}}{r}\frac{\partial\overline{ww}}{\partial r} + 4\overline{ww}(\overline{v^2}-\overline{w^2})\right] - \frac{3}{2}\frac{\varepsilon}{k}\overline{v^2}+\frac{\varepsilon}{3} \tag{6.21}$$

$$U_o\frac{\partial\overline{w^2}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[.11r\frac{k}{\varepsilon}\overline{vv}\frac{\partial\overline{w^2}}{\partial r}\right] + .11\frac{k}{\varepsilon}\left[2\frac{\overline{vv}}{r}\frac{\partial\overline{ww}}{\partial r} + 4\overline{ww}\frac{(\overline{v^2}-\overline{w^2})}{r^2}\right]$$

$$+ \frac{2}{r}\frac{\partial}{\partial r}\left[.11\frac{k\overline{ww}}{\varepsilon}(\overline{vv}-\overline{ww})\right] - \frac{3}{2}\frac{\varepsilon}{k}\overline{ww}+\frac{\varepsilon}{3} \tag{6.22}$$

$$U_o\frac{\partial\varepsilon}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[.165r\frac{k}{\varepsilon}\overline{vv}\frac{\partial\varepsilon}{\partial r}\right] - 1.9\frac{\varepsilon^2}{k} \tag{6.23}$$

$$U_o\frac{\partial\overline{uv}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[.11\frac{rk}{\varepsilon}\left(2\overline{v^2}\frac{\partial\overline{uv}}{\partial r} + \overline{uv}\frac{\partial\overline{v^2}}{\partial r}\right)\right] - .11\frac{k}{\varepsilon}\left[\frac{\overline{uv}}{r}\frac{\partial\overline{ww}}{\partial r} + 2\overline{w^2}\frac{\overline{uv}}{r^2}\right]$$

$$- \frac{(2.6\overline{v^2}+2k-1.2\overline{u^2})}{11}\frac{\partial U_d}{\partial r} - \frac{3}{2}\frac{\varepsilon}{k}\overline{uv} \tag{6.24}$$

$$U_o\frac{\partial U_d}{\partial x} = -\frac{1}{r}\frac{\partial}{\partial r}[-\overline{uv}r] \tag{6.25}$$

(For the algebraic details of the derivation, see Appendix).

Again, there is no tendency to isotropy, as $\overline{u^2}$ $\overline{v^2}$ and $\overline{w^2}$ tend to different values as the self-similar solution is approached.

Seeking self-similar solutions of the type:

$$k = Kh(\eta) \quad \overline{u^2} = K\phi(\eta) \quad \overline{v^2} = K\chi(\eta) \quad \overline{w^2} = K\psi(\eta) \quad U_d = U_D f(\eta)$$

$$\overline{uv} = K^{1/2}U_D g(\eta) \;,$$

one obtains the following equation for ϕ, χ and η :

$$\frac{1}{\eta}\frac{d}{d\eta}[.11\eta\frac{\chi h}{e}\frac{d\phi}{d\eta}] + \frac{U_oK}{EL}\frac{dL}{dx}\eta\phi' - \frac{3}{2}\frac{e}{h}\phi + \frac{e}{3} - \frac{U_o}{E}\frac{dK}{dx}\phi = 0 \tag{6.26}$$

$$\frac{1}{\eta}\frac{d}{d\eta}[.33\eta\frac{\chi h}{e}\frac{d\chi}{d\eta}] + \frac{U_o}{E}\frac{K}{L}\frac{dL}{dx}\eta\chi' - .11\frac{h}{e}[\frac{2\chi}{\eta}\psi' + \frac{4\psi(\chi-\psi)}{\eta^2}] - \frac{3}{2}\frac{e}{h}\chi + \frac{e}{3} - \frac{U_o}{E}\frac{dK}{dx}\chi = 0 \tag{6.27}$$

$$\frac{1}{\eta}\frac{d}{d\eta}[.11\frac{h}{e}(\eta\chi\frac{d\psi}{d\eta} + 2\psi(\chi-\psi))] + .11\frac{h}{e}[\frac{2\chi\psi'}{\eta} + \frac{4\psi(\chi-\psi)}{\eta^2}] + \frac{U_oK}{EL}\frac{dL}{dx}\eta\psi' - \frac{3}{2}\frac{e}{h}\psi$$

$$+\frac{e}{3} + \frac{U_o}{E}\frac{dK}{dx}\psi = 0 \tag{6.28}$$

$$\frac{1}{\eta}\frac{d}{d\eta}[.165\eta\frac{h\chi}{e}\frac{de}{d\eta}] + \frac{U_oK}{EL}\frac{dL}{dx}\eta e' - 1.9\frac{e^2}{h} + \frac{KU_o}{E^2}\frac{dE}{dx}e = 0 \tag{6.29}$$

The solution of the above equations gives:

$$\frac{KU_o}{EL}\frac{dL}{dx} = .225 \qquad \frac{U_o}{E}\frac{dK}{dx} = 1.48 \tag{6.30}$$

and

$$n_k = -1.53 \qquad n_L = .233 \tag{6.31}$$

The h, ϕ, χ and ψ profiles are shown in Figure 9.

Naudascher seems to suggest that in the asymptotic state $\overline{u^2} = \overline{v^2} = \overline{w^2}$. However, the computed difference of the values of these quantities at the centerline is of the same order of magnitude as the error in Naudascher's results, and it is not possible to disqualify this model by virtue of this part of the Naudascher data.

Naudascher shows distribution of $\overline{u^2}$, but not $\overline{v^2}$ and $\overline{w^2}$. On the other hand, he also shows the distribution of P, which, in the boundary layer approximation is related to $\overline{v^2}$ and $\overline{w^2}$ by the following equation (see Townsend).

$$\frac{P}{\rho} + \overline{v^2} + \int_0^r \frac{\overline{v^2} - \overline{w^2}}{r}\,dr = \text{Const.} \tag{6.32}$$

P does not seem to tend to its freestream value slower than $\overline{u^2}$ in Naudascher's paper. As however, P was measured by pressure probes and $\overline{u^2}$ by hot wire anemometers, it is not sure that these quantities are comparable, and it is thought premature to disqualify this model by virtue of this aspect (i.e. non tendency to isotropy).

The momentum and $\overline{uv}$ equation are:

$$\frac{d}{d\eta}[.11\frac{h}{e}(2\chi\frac{dg}{d\eta} + g\frac{d\chi}{d\eta})] + .11\frac{h}{e}[\frac{g}{\eta}\frac{d}{d\eta}(\chi - \psi) + \frac{2}{\eta}\chi g' - \frac{2g\psi}{\eta^2}] - (\frac{U_oK}{EU_D}\frac{dU_D}{dx} + \frac{U_o}{2E}\frac{dK}{dx})g$$

$$+ \frac{K}{LE}\frac{dL}{dx}\eta g' - \frac{(2.6\chi + 2h - 1.2\phi)}{11}f' - \frac{3}{2}\frac{e}{h}g = 0 \tag{6.33}$$

$$-\frac{U_oK}{EU_D}\frac{dU_D}{dx}f + \frac{U_oK}{LE}\frac{dL}{dx}\eta f' - \frac{1}{\eta}\frac{d}{d\eta}(g\eta) = 0 \tag{6.34}$$

whose solution is:

$$\frac{U_oK}{U_DE}\frac{dU_D}{dx} = -.87\ , \quad n_u = -.9 \tag{6.35}$$

The f and g distributions are shown in Figure 10.

Again, the value of n_u is too low, and this is completely contradicted by Naudascher's results, in which $n_\eta \simeq n_k \simeq -1.5$. Again, this is a result of the $\overline{uv}\frac{\overline{dvv}}{dy}$ term in the $\overline{uv}$ equation; without this term, the results of the model would be much better, but obviously consistency will be lost.

7. Comparison with Experimental Results

7.1 Integral Parameters

It is first necessary to convert the integral parameters to Naudascher's notation. This author uses as length scale $r_{1/2}$, i.e. the value of axial coordinate at which k = .25K. From Figure 6 it is seen that $r_{1/2}$ = 1.7L. As a turbulent scale, he uses $u' = \sqrt{2k/3}$.

Thus:

$$\frac{U_o}{u'}\frac{dr_{1/2}}{dx} = \frac{1.7}{\sqrt{2/3}}\frac{1}{K^{1/2}}\frac{dL}{dx} = 2.08\frac{EL}{K^{3/2}}\frac{K}{EL}\frac{dL}{dx} = 2.08\frac{K}{EL}\frac{dL}{dx} \tag{7.1}$$

$$\frac{\frac{1}{r_{1/2}}\frac{dr_{1/2}}{dx}}{\frac{1}{U_D}\frac{dU_D}{dx}} = \frac{\frac{U_oK}{EL}\frac{dL}{dx}}{\frac{U_oK}{EU_D}\frac{dU_D}{dx}} \tag{7.2}$$

$$\frac{\frac{1}{r_{1/2}}\frac{dr_{1/2}}{dx}}{\frac{1}{\overline{u'^2}}\frac{d\overline{u'^2}}{dx}} = \frac{\frac{U_oK}{EL}\frac{dL}{dx}}{\frac{U_o}{E}\frac{dK}{dx}} \tag{7.3}$$

$$\frac{U_oU_D}{\overline{u'v'}_{max}}\frac{dr_{1/2}}{dx} = \frac{U_o 1.7U_D}{.2U_DK^{1/2}}\frac{dL}{dx} = 8.5\frac{K}{EL}\frac{dL}{dx} \tag{7.4}$$

Table I

Parameter		7.1	7.2	7.3	7.4
Calculated, k-ε model		.458	-.212	-.183	1.87
Naudascher, $\frac{x}{D}$ =	5	.48	-.36	-.36	3
	7	.51	-.26	-.26	2.4
	10	.54	-.2	-.2	1.8
	15	.4	-.14	-.14	1.14
	20	.41	-.12	-.12	1.09
	25	.47	-.12	-.12	1.02

In Table I, the values of the parameters listed were calculated by Naudascher. The values of the parameters calculated by the LI and LII aren't listed, because these models do not compare as well with the data as the k-ε model.

It is seen from Table I, parameter (7.1), that the predicted rate of increase of the length scale is in excellent agreement with the data.

Although the computed value of parameter 7.3 is not in very good agreement with the values computed by Naudascher, it can be seen from Figure 11 that the slope of the power law, $n_k = -1.53$, compares well with our predicted value of the slope, $n_k = -1.46$. In Figure 11a, it is seen that the power law slope is $n_k = -1.5$. This shows that the results of Lin & Pao and Gran are also in good agreement with our model.

Our prediction of parameter 7.4 is also not in very good agreement with Naudascher's results; however, $\overline{uv}_{max}$ is involved in this parameter. The absolute error in this quantity is of the same order of magnitude as the error in the values of $\overline{u^2}$ and $\overline{v^2}$, which are many times larger than $\overline{uv}$. From Table I in Naudascher, one can estimate the error in $\overline{v^2}$ by setting it equal to $\overline{v^2}_{max} - \overline{w^2}_{max}$ (which, for circular geometry, should be zero). It is seen that the accuracy of $\overline{uv}_{max}$ is $\pm$ 100%, and therefore even the order of magnitude agreement, shown in Table I, is satifactory.

The most unsatisfactory feature of this model seems to be the poor agreement of parameter (7.2) with the value predicted by Naudascher. Naudascher suggests that $\overline{u^2}$ decays as U_D, whereas the k-ε model suggests U_D exhibits a slightly slower rate of decay. Apparently, this is what lead Finson to postulate that U_D decays as $\overline{u^2} - \overline{v^2}$.

On the other hand, the data of Lin & Pao (Figure 11a) show that U_D decays actually less fast than $\overline{u^2}$, and predicts a slope of -1, instead of -1.27 (as the k-ε model suggests). Notice, however, that the spread in their data is rather too big to discriminate between these two values.

Finally, it is to be noted that (LI and especially LII) are worse than the k-ε model.

In Figures 12, 12a, 13 and 13a, the turbulent energy and velocity distribution as predicted by the k-ε model, are compared with the data of Naudascher, Lin & Pao and Gran. Fair agreement is shown.

In conclusion, one must state that it is rather unfortunate that all higher-order models, in spite of their complexity, show a poorer performance in comparison to the simple k-ε gradient model. It is thought that this model might be improved by slightly modifying the diffusion constants.

APPENDIX

Reynolds Stress Diffusion According to LII Model

The purpose of this appendix is to calculate the components of $\Lambda^{ijk}_{,k}$, where:

$$\Lambda^{ijk} = \tau(\overline{u^i u^m}\ \overline{u^j u^k_{,m}} + \overline{u^j u^m}\ \overline{u^i u^k_{,m}} + \overline{u^k u^m}\ \overline{u^i u^j_{,m}}) \qquad \text{(A.1)}$$

in axisymmetric cylindrical geometry without swirl.

Note that the only non-vanishing components of the Christofel symbol in cylindrical coordinates are:

$$\Gamma^r_{\theta\theta} = -r \quad \Gamma^\theta_{\theta r} = \Gamma^\theta_{r\theta} = \frac{1}{r}$$

$$\Lambda^{zzk}_{,k} = \frac{\partial \Lambda^{zzr}}{\partial r} + \Gamma^k_{mk}\Lambda^{zzm} = \frac{\partial \Lambda^{zzr}}{\partial r} + \frac{\Lambda^{zz\theta}}{r}$$

$$\Lambda^{zzr} = \tau(\overline{u^r u^r}\ \overline{u^z u^z_{,r}} + 2\overline{u^z u^r}\ \overline{u^z u^r_{,r}})$$

$$\Lambda^{zz\theta} = 0$$

$$\overline{u^z u^z_{,r}} = \frac{\partial \overline{u^z u^z}}{\partial r} \qquad \overline{u^z u^r_{,r}} = \frac{\partial \overline{u^z u^r}}{\partial r}$$

$$\Lambda^{zzk}_{,k} = \frac{1}{r}\frac{\partial}{\partial r}[r\tau\{\overline{u^r u^r}\ \frac{\partial \overline{u^z u^z}}{\partial r} + 2\overline{u^z u^r}\ \frac{\partial \overline{u^z u^r}}{\partial r}\}] \qquad \text{(A.2)}$$

$$\Lambda^{rrk}_{,k} = \frac{\partial \Lambda^{rrr}}{\partial r} + 2\Gamma^r_{mk}\Lambda^{mrk} + \Gamma^k_{mk}\Lambda^{rrm}$$

$$= \frac{\partial \Lambda^{rrr}}{\partial r} + \frac{\Lambda^{rrr}}{r} - 2r\Lambda^{\theta r\theta}$$

$$\Lambda^{rrr} = 3\tau\overline{u^r u^r}\ \overline{u^r u^r_{,r}} = 3\tau(\overline{u^r u^r}\ \frac{\partial \overline{u^r u^r}}{\partial r})$$

$$\Lambda^{\theta r\theta} = \tau(2\overline{u^\theta u^\theta}\ \overline{u^r u^\theta_{,\theta}} + \overline{u^r u^r}\ \overline{u^\theta u^\theta_{,r}})$$

$$= \tau[\overline{u^\theta u^\theta}(\Gamma^r_{\theta\theta}\overline{u^\theta u^\theta} + \Gamma^\theta_{r\theta}\overline{u^r u^r}) + \overline{u^r u^r}(\frac{\partial \overline{u^\theta u^\theta}}{\partial r} + 2\Gamma^\theta_{\theta r}\overline{u^\theta u^\theta})]$$

$$= 2\tau[\frac{\overline{ww}}{r^2}\{-r\frac{\overline{ww}}{r^2} + \frac{\overline{vv}}{r}\} + \frac{\overline{vv}}{r^2}\frac{\partial\overline{w^2}}{\partial r}]$$

$$= 2\tau\frac{\overline{ww}}{r^3}(\overline{vv} - \overline{ww}) + \tau\frac{\overline{vv}}{r^2}\frac{\partial\overline{ww}}{\partial r}$$

(Use has been made of the relation $\overline{ww} = r^2\overline{u^\theta u^\theta}$)

$$\Lambda^{rrk}_{,k} = \frac{1}{r}\frac{\partial}{\partial r}[\tau r 3\overline{vv}\frac{\partial\overline{vv}}{\partial r}] - 2\tau\frac{\overline{vv}}{r}\frac{\partial\overline{ww}}{\partial r} - 4\tau\frac{\overline{ww}}{r^2}(\overline{v^2} - \overline{w^2}) \tag{A.3}$$

$$\Lambda^{\theta\theta k}_{,k} = \frac{\partial\Lambda^{\theta\theta r}}{\partial r} + \Gamma^k_{mk}\Lambda^{\theta\theta m} + 2\Gamma^\theta_{mk}\Lambda^{m\theta k}$$

$$= \frac{\partial\Lambda^{\theta\theta r}}{\partial r} + \Gamma^\theta_{r\theta}\Lambda^{\theta\theta r} + 4\Gamma^\theta_{\theta r}\Lambda^{\theta\theta r}$$

$$= \frac{\partial\Lambda^{\theta\theta r}}{\partial r} + 5\Gamma^\theta_{r\theta}\Lambda^{\theta\theta r} = \frac{1}{r^5}\frac{\partial}{\partial r}[r^5\Lambda^{\theta\theta r}]$$

$$\Lambda^{\theta\theta r} = \tau(2\overline{u^\theta u^\theta}\ \overline{u^\theta u^r_{,\theta}} + \overline{u^r u^r}\ \overline{u^\theta u^\theta_{,r}})$$

$$= \tau[2\overline{u^\theta u^\theta}(\Gamma^\theta_{r\theta}\overline{u^r u^r} + \Gamma^r_{\theta\theta}\overline{u^\theta u^\theta}) + \overline{u^r u^r}(\frac{\partial\overline{u^\theta u^\theta}}{\partial r} + 2\Gamma^\theta_{r\theta}\overline{u^\theta u^\theta})]$$

$$= \tau[2\frac{\overline{ww}}{r^3}(\overline{v^2} - \overline{w^2}) + \frac{\overline{vv}}{r^2}\frac{\partial\overline{ww}}{\partial r}]$$

$$\Lambda^{\theta\theta k}_{,k} = \frac{1}{r^5}\frac{\partial}{\partial r}[\tau r^3\overline{vv}\frac{\partial\overline{ww}}{\partial r} + 2r^2\tau\,\overline{ww}(\overline{v^2} - \overline{w^2})]$$

$$= \frac{1}{r^2}[\frac{1}{r}\frac{\partial}{\partial r}(r\tau\overline{v^2}\frac{\partial\overline{w^2}}{\partial r}) + \frac{2\overline{v^2}\tau}{r}\frac{\partial\overline{w^2}}{\partial r} + \frac{4\tau\overline{w^2}(\overline{v^2} - \overline{w^2})}{r^2} + \frac{2}{r}\frac{\partial}{\partial r}\{\tau\overline{w^2}(\overline{v^2} - \overline{w^2})\}] \tag{A.4}$$

$$\Lambda^{zrk}_{,k} = \frac{\partial\Lambda^{zrr}}{\partial r} + \Gamma^r_{mk}\Lambda^{zmk} + \Gamma^k_{mk}\Lambda^{zrm}$$

$$= \frac{\partial\Lambda^{zrr}}{\partial r} + \Gamma^r_{\theta\theta}\Lambda^{z\theta\theta} + \Gamma^\theta_{r\theta}\Lambda^{zrr}$$

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$$\Lambda^{zrr} = \tau(2\overline{u^r u^r}\ \overline{u^z u^r_{,r}} + 2\overline{u^z u^r}\ \overline{u^r u^r_{,r}})$$

$$= \tau(2\overline{u^r u^r}\ \frac{\partial \overline{u^z u^r}}{\partial r} + \overline{u^z u^r}\ \frac{\partial \overline{u^r u^r}}{\partial r})$$

$$\Lambda^{z\theta\theta} = \tau(\overline{u^z u^r}\ \overline{u^\theta u^\theta_{,r}} + 2\overline{u^\theta u^\theta}\ \overline{u^z u^\theta_{,\theta}})$$

$$= \frac{\tau}{r^2}(\overline{uv}\ \frac{\partial \overline{w^2}}{\partial r} + 2\overline{w^2}\ \frac{\overline{uv}}{r})$$

$$\Lambda^{zrk}_{,k} = \frac{1}{r}\frac{\partial}{\partial r}[\tau r(2\overline{v^2}\ \frac{\partial \overline{uv}}{\partial r} + \overline{uv}\ \frac{\partial \overline{v^2}}{\partial r})] - (\frac{\overline{uv}}{r}\ \frac{\partial \overline{ww}}{\partial r} + 2\frac{\overline{ww}}{r^2}\ \overline{uv})\tau \qquad (A.5)$$

$$= \frac{\partial}{\partial r}[\tau(2\overline{v^2}\ \frac{\partial \overline{uv}}{\partial r} + \overline{uv}\ \frac{\partial \overline{v^2}}{\partial r})] + \tau\frac{\overline{uv}}{r}\ \frac{\partial(\overline{v^2} - \overline{w^2})}{\partial r} + \frac{2\tau}{r}(\overline{v^2}\ \frac{\partial \overline{uv}}{\partial r} - \overline{w^2}\ \frac{\overline{uv}}{r^2}) \qquad (A.6)$$

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LIST OF SYMBOLS

C_D	Drag coefficient of body
e	Non-dimensionalized turbulent energy dissipation $(= \frac{\varepsilon}{E})$
E	Value of turbulent energy dissipation at the centerline
f	Non-dimensionalized velocity defect $(= U_d/U_D)$
g	Non-dimensionalized Reynolds stress $(= \overline{uv}/U_D K^{1/2})$
h	Non-dimensionalized turbulent energy $(= k/K)$
K	Value of turbulent energy at centerline
L	Characteristic length scale of wake
L_m	Macroscale of turbulence
P	Pressure
P_∞	Pressure at the free stream
P_{min}	Pressure at the centerline
p	Non-dimensionalized pressure (P/P_{min})
r	Radial Coordinate
$r_{1/2}$	Value of radial coordiante where $k = K/4$
U_o	Free stream velocity
U_d	Velocity defect
U_D	Velocity defect at centerline
u,v,w	Fluctuating components of velocity
W	Swirl velocity
x,y,z	Cartesian coordinate
ε	Turbulent energy dissipation rate
θ	Momentum thickness
Λ	Characteristic length scale of large eddies
λ	Microscale of turbulence

ν	Kinematic viscosity
ν_T	Eddy diffusivity
ρ	Density
ϕ	$= \overline{u^2}/K$
χ	$= \overline{v^2}/K$
ψ	$= \overline{w^2}/K$

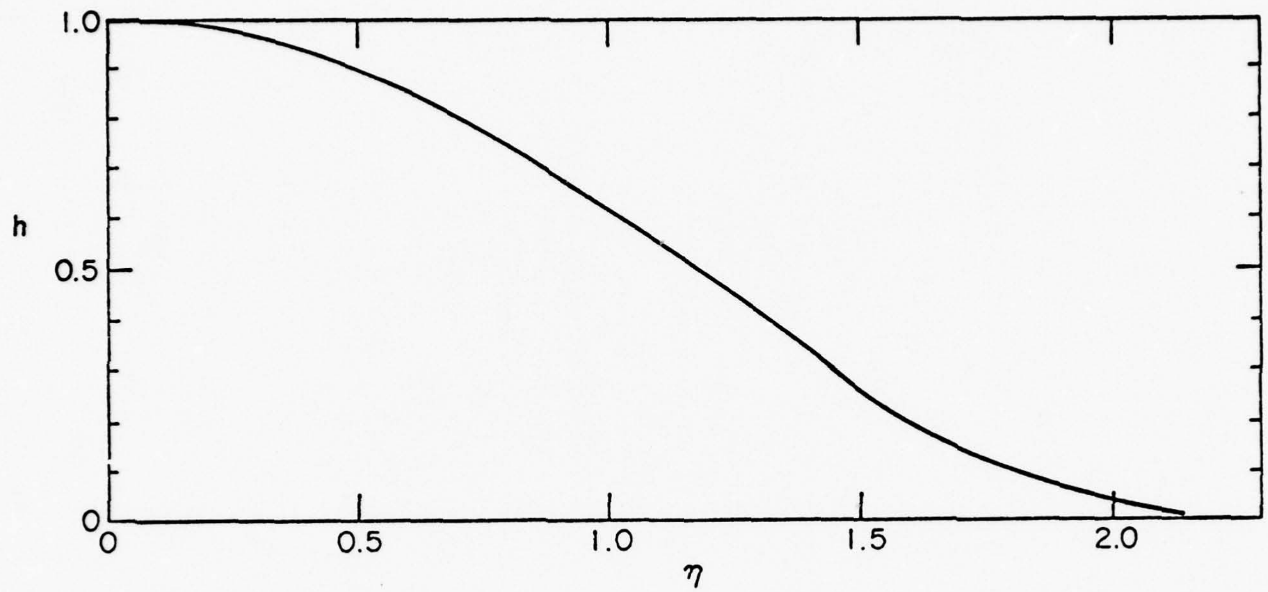

Figure 1. Turbulent Energy Profile Calculated by k-ε Model (Plane Geometry)

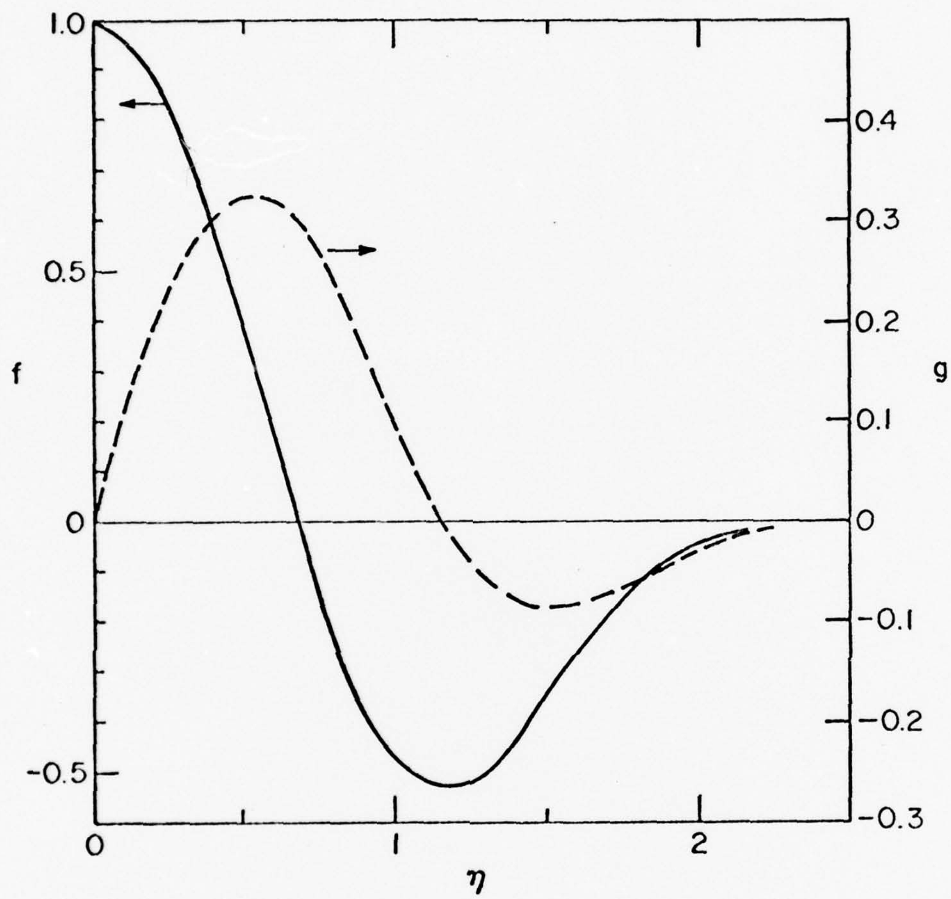

Figure 2. Self Similar Velocity (f) and Reynolds Stress (g) Profiles Calculated by k-ε Model (Plane Geometry)

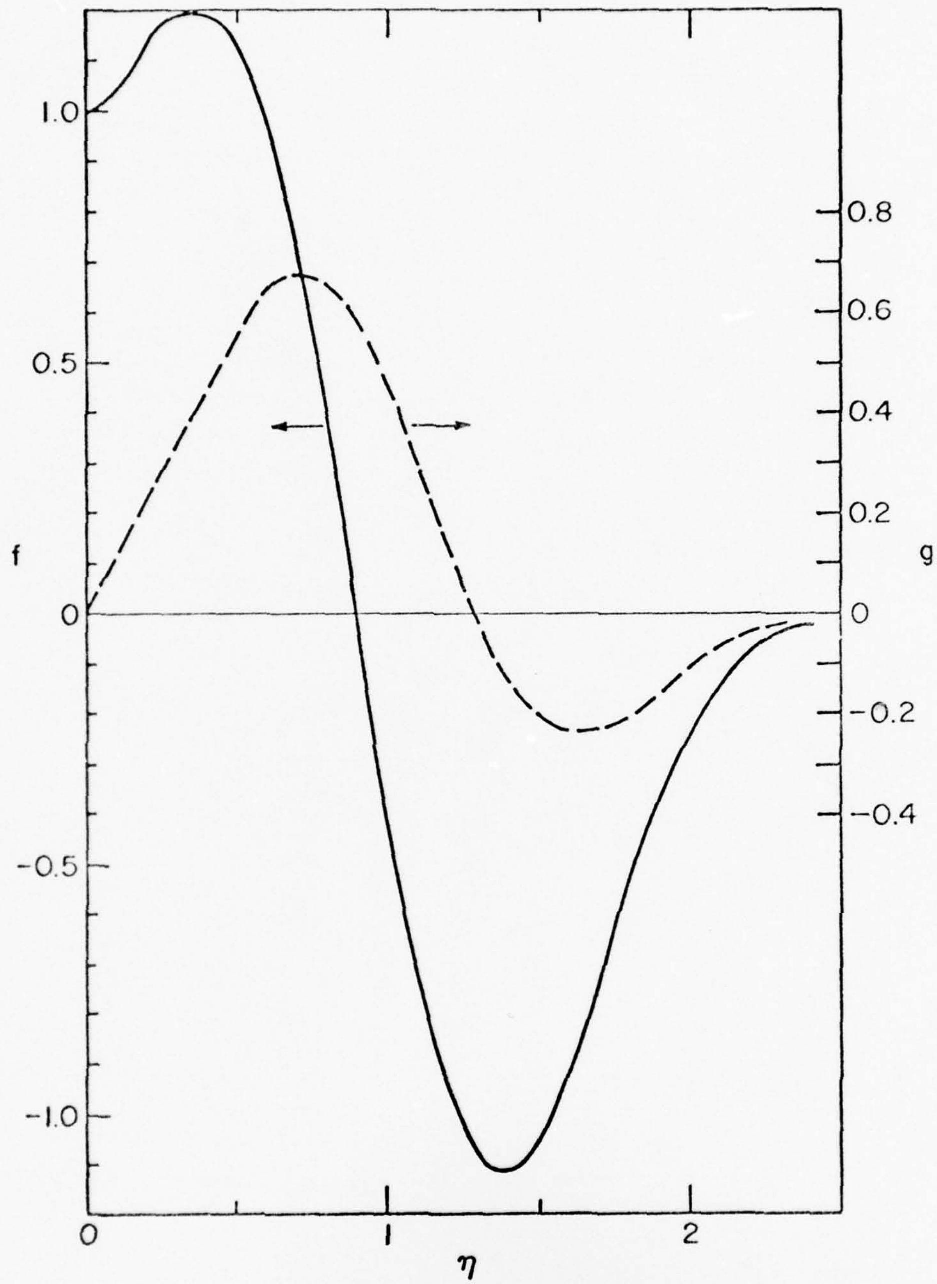

Figure 3. Self Similar Velocity (f) and Reynolds Stress (g) Profiles Calculated by LI Model (Plane Geometry)

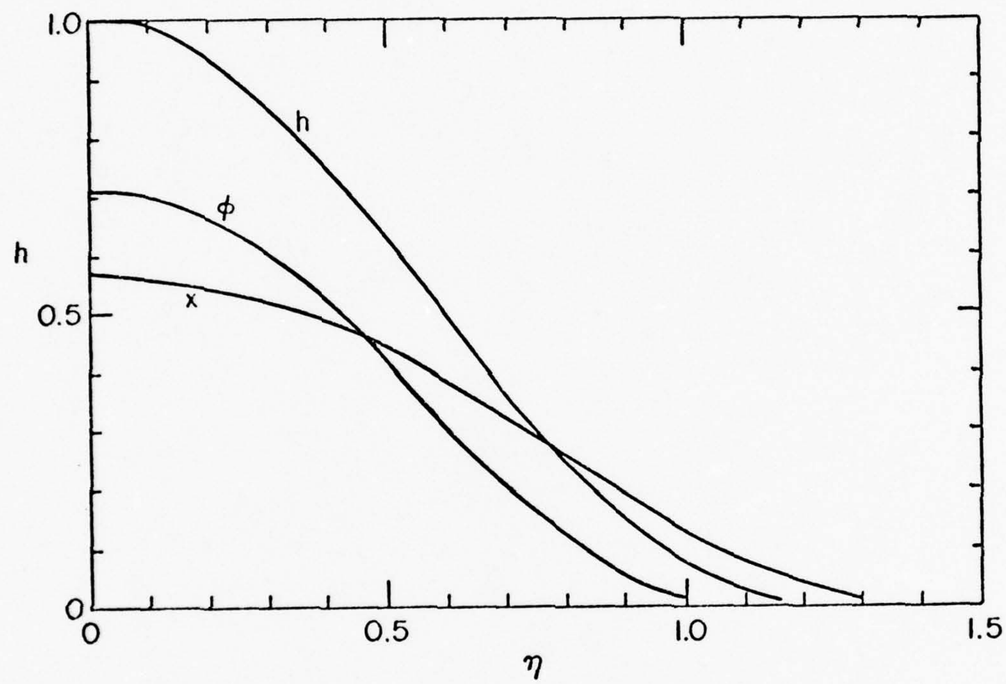

Figure 4. Profiles of Turbulent Energy ($h = k/K_o$), Streamwise Component of Reynolds Stress ($\phi = u^2/K_o$) and Cross-Stream Component of Reynolds Stress ($\chi = v^2/K_o$) (Plane Geometry) Calculated by LII Model (Plane Geometry)

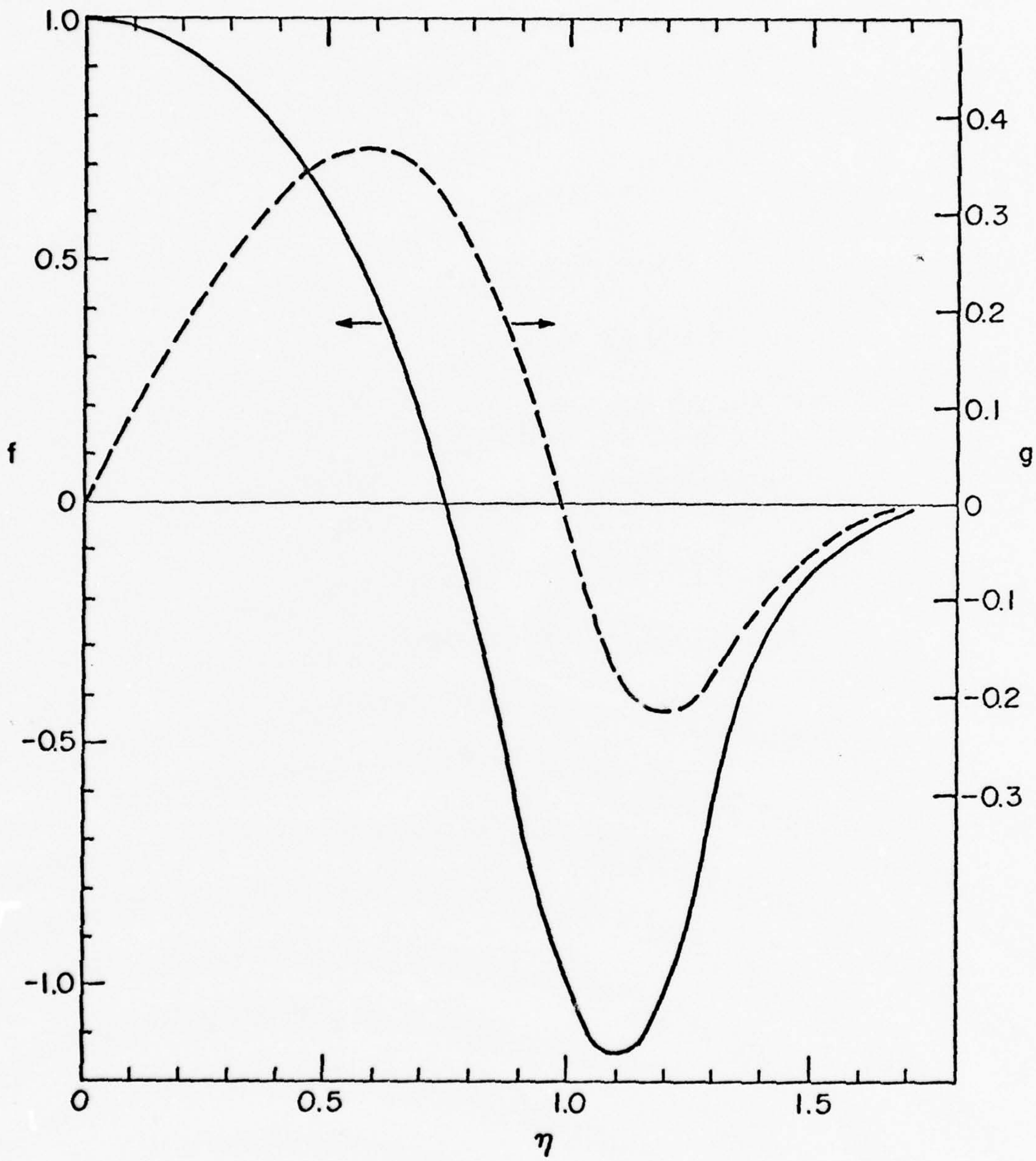

Figure 5. Velocity Defect and Reynolds Stress Distribution for Momentumless Plane Wake (Model LII)

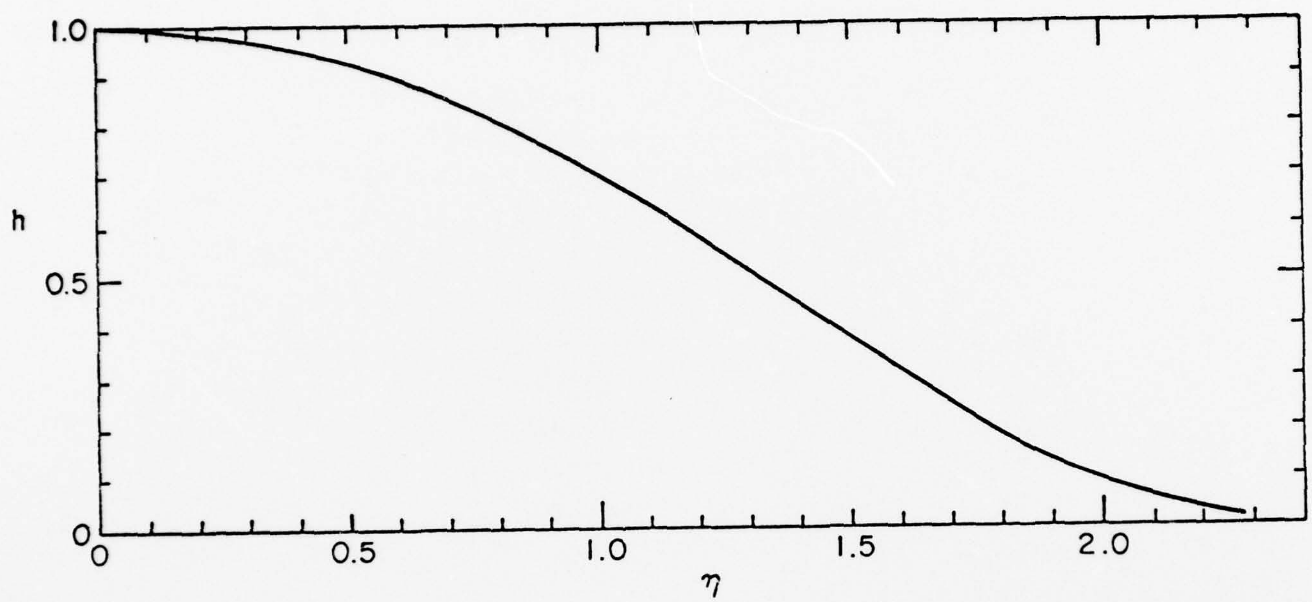

Figure 6. Turbulent Energy Distribution Calculated by LI Model (Axisymmetric Geometry)

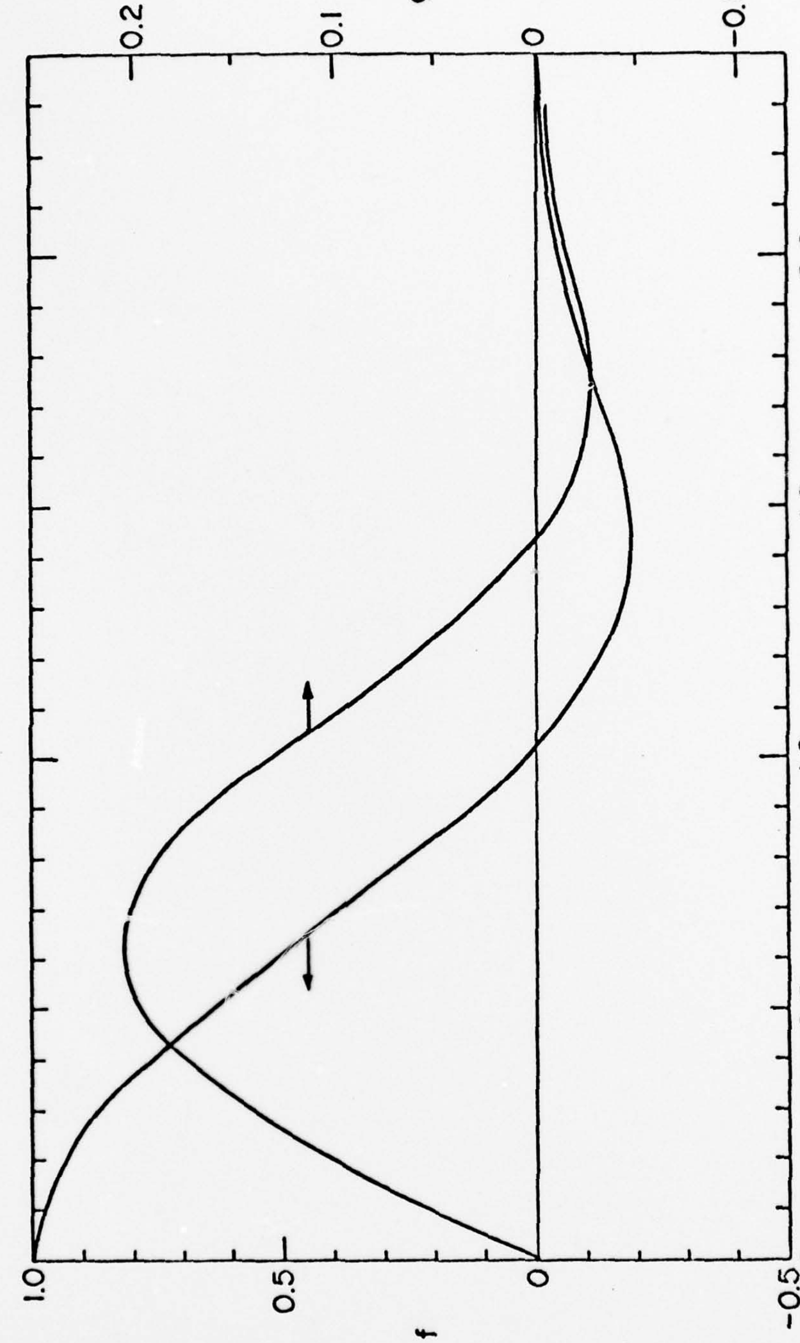

Figure 7. Velocity Defect and Reynolds Stress Distribution for Axisymmetric Momentumless Wakes as Calculated by k-ε Model

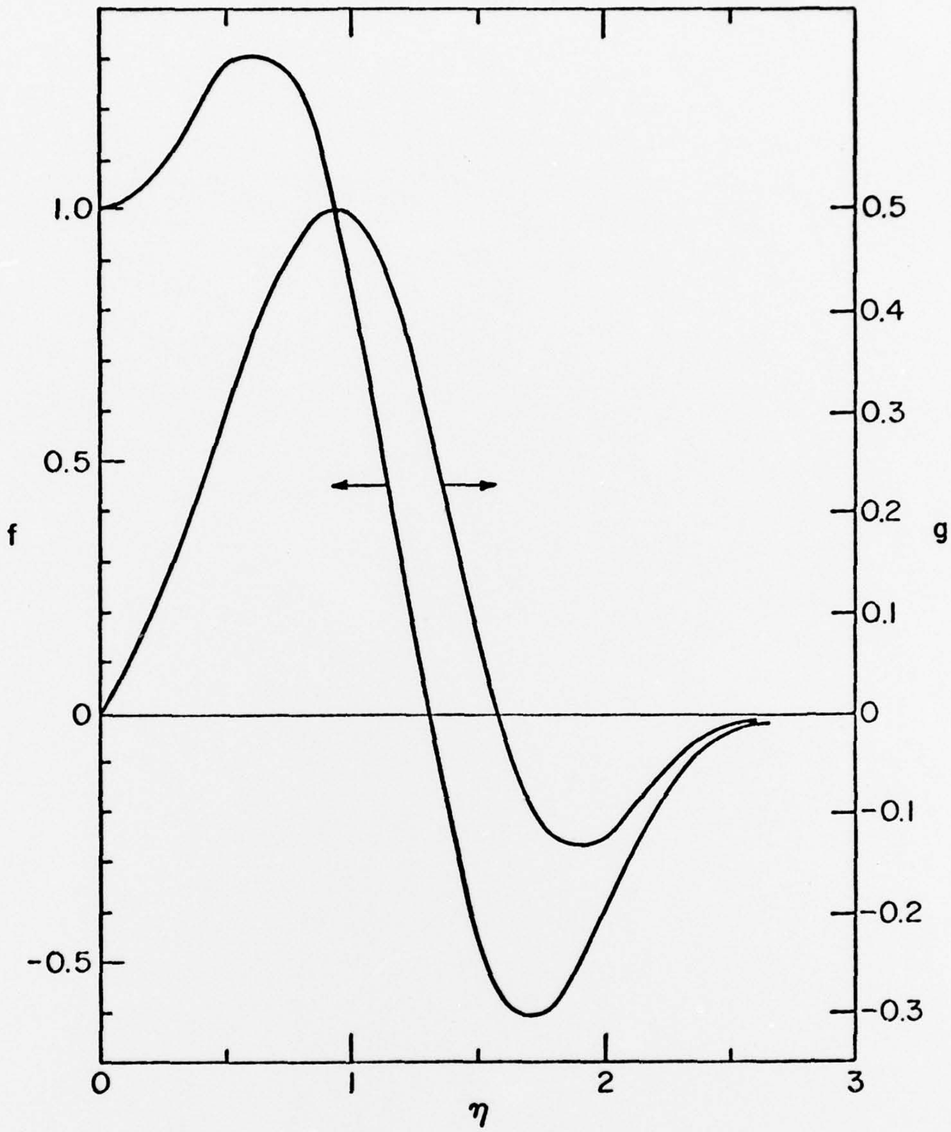

Figure 8. Velocity Defect Distribution and Reynolds Stress for Axisymmetric Momentumless Wake Calculated by LI Model

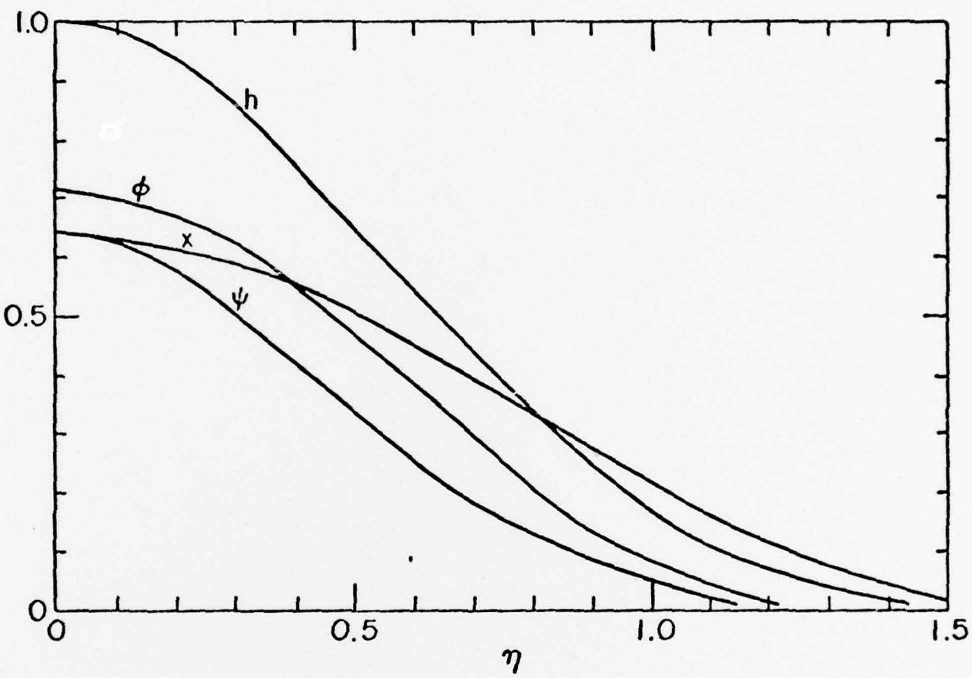

Figure 9. Turbulent Energy (h) and $\overline{uu}$, $\overline{vv}$, $\overline{ww}$ Correlations (ϕ,χ,ψ) for Momentumless Self Similar Axisymmetric Wake Calculated by LII Model

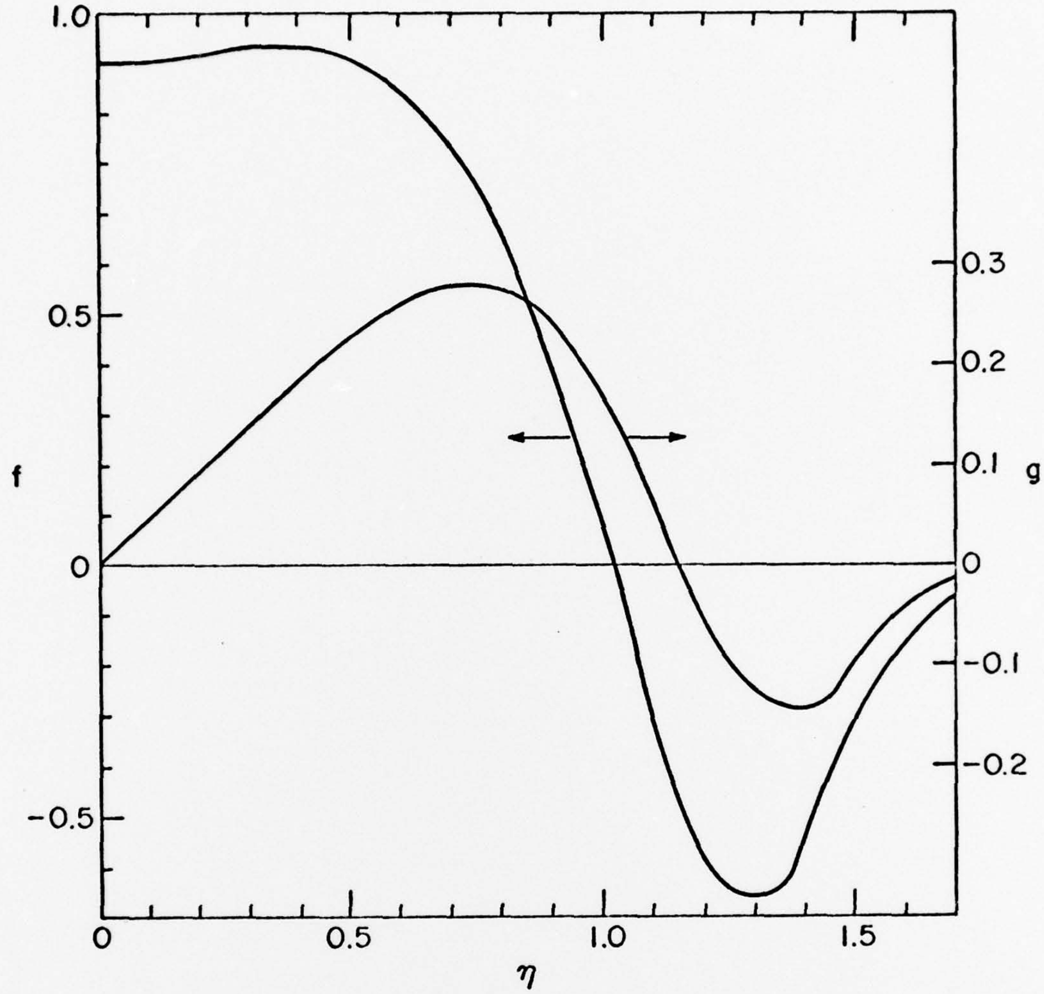

Figure 10. Velocity Defect and Reynolds Shear Stress Distribution in Axisymmetric Momentumless Wake as Calculated by LII Model

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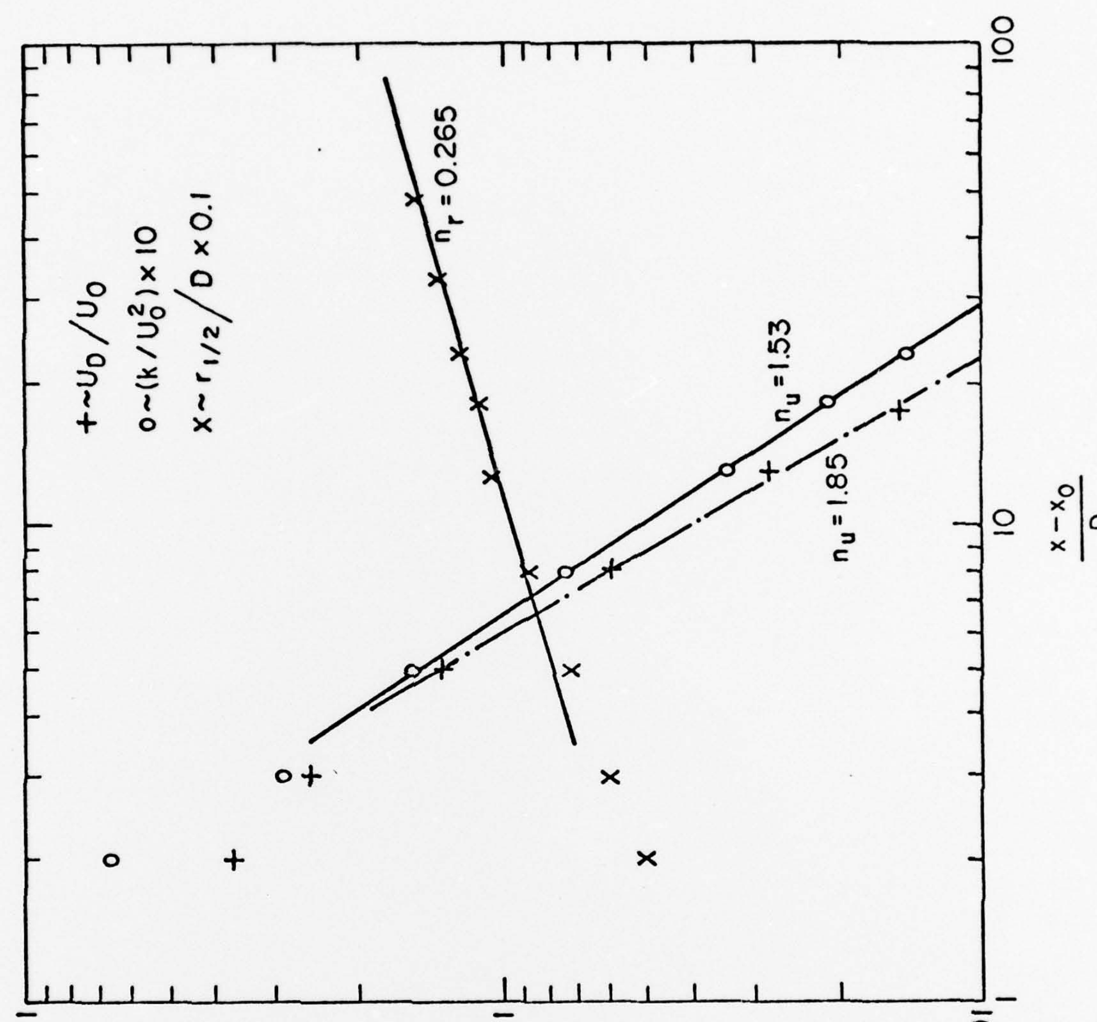

Figure 11. Decay of U_D, k and $r_{1/2}$ in Naudascher's Results

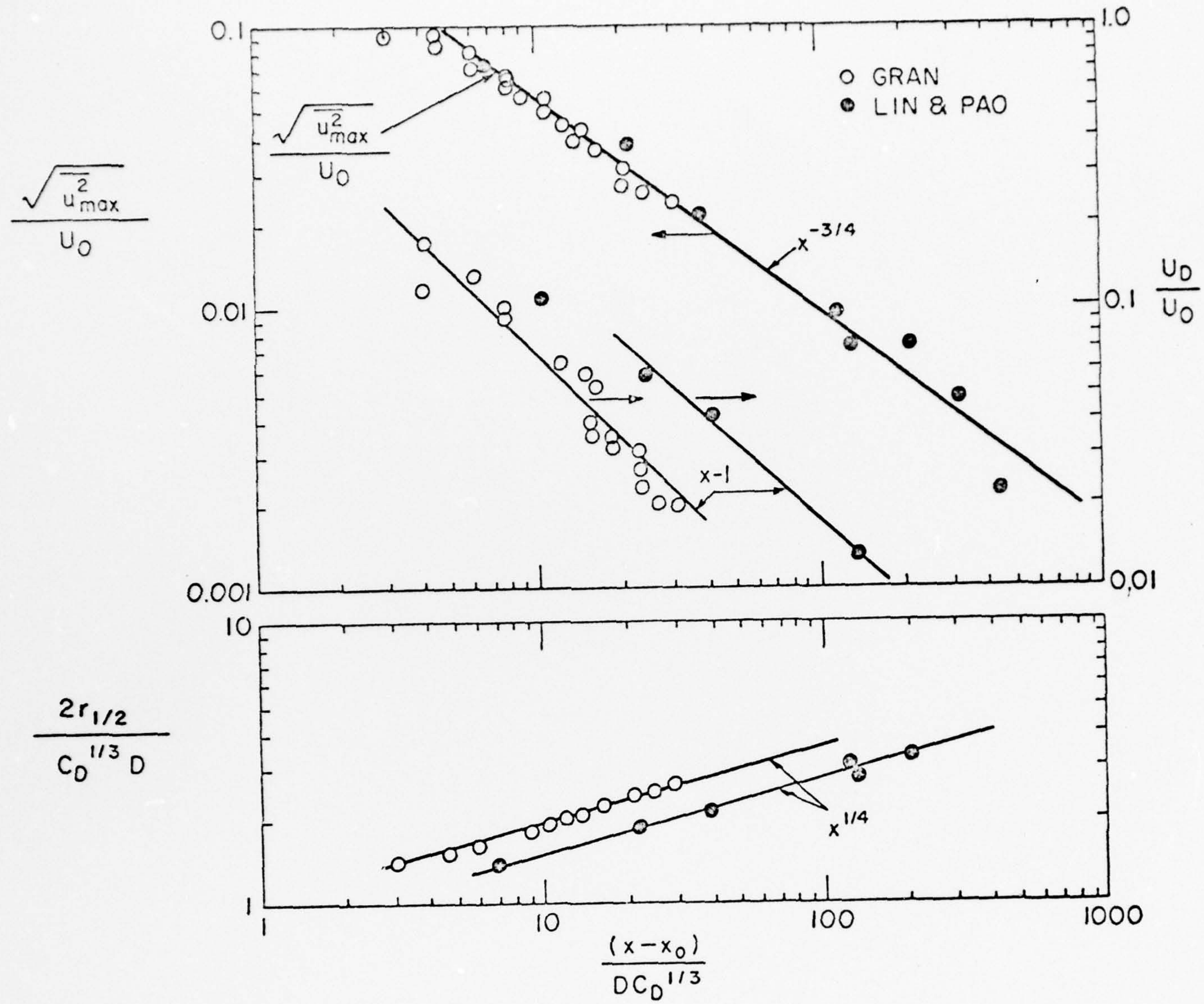

Figure 11a. $\frac{\sqrt{\overline{u^2_{max}}}}{U_o}$, $\frac{U_D}{U_o}$ and $\frac{r_{1/2}}{CD^{1/2}D/2}$ Growth in Propeller Driven Momentumless Wakes as Measured by Gran (1973) and Lin & Pao (1974)

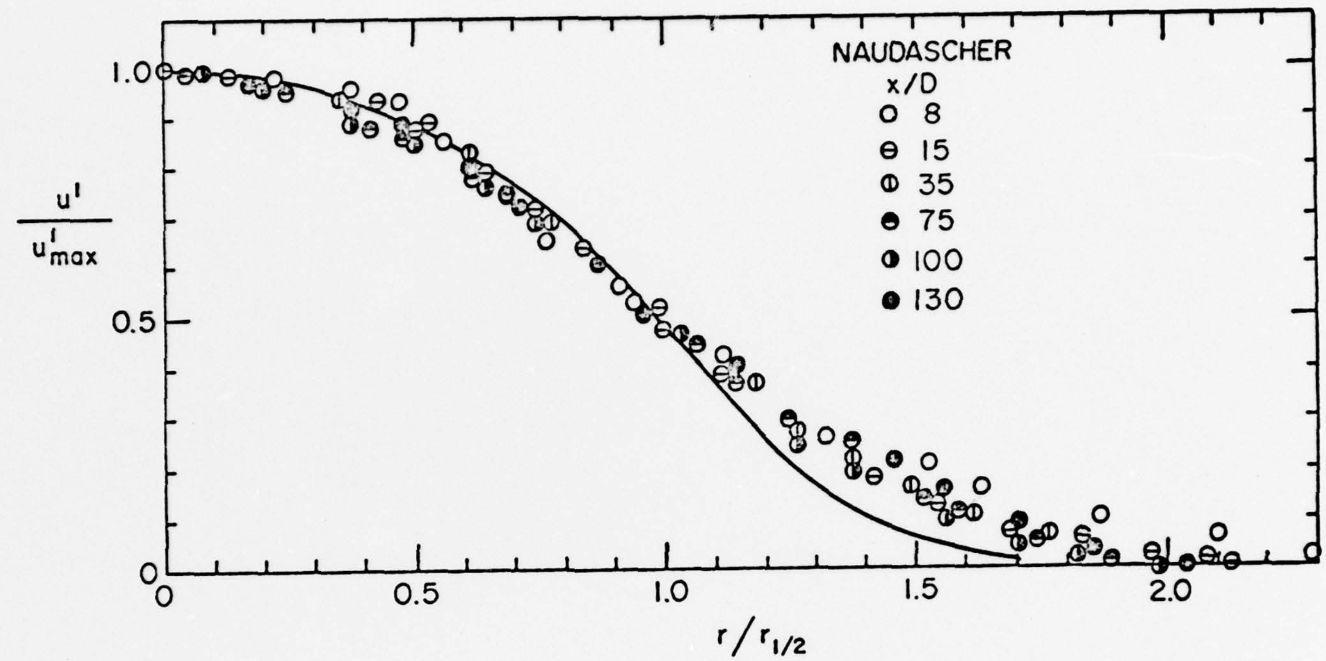

Figure 12. Comparison of Measured Axial r.m.s. Velocity Distribution with Prediction of (k-ε) Model

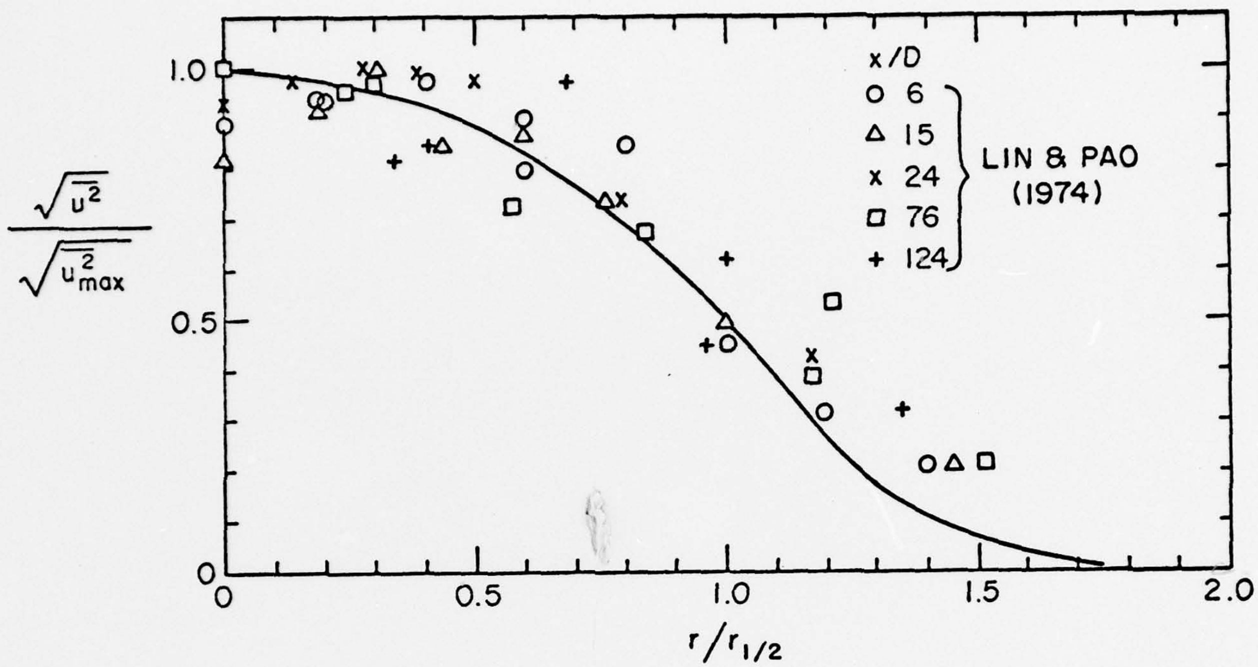

Figure 12a. Comparison of Measured Axial Fluctuating Velocity Distribution with Predictions of the k-ε Model

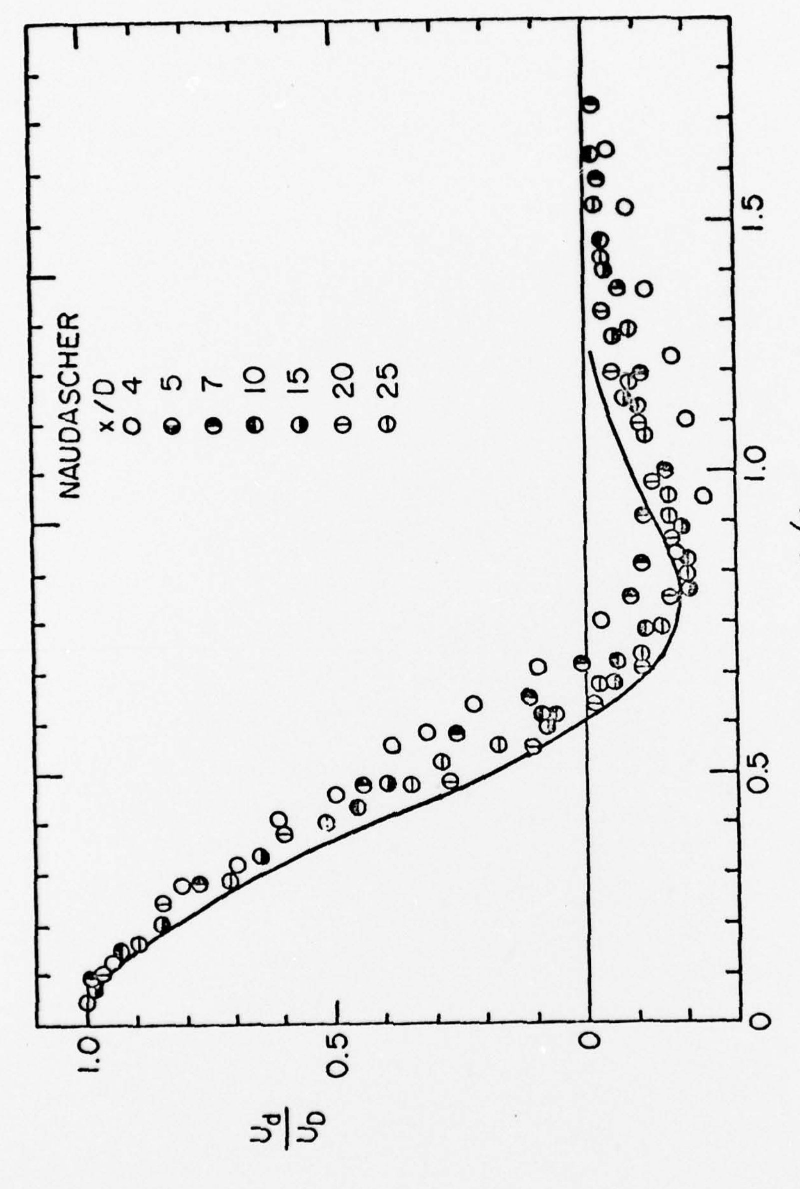

Figure 13. Self-Similar Fluctuating Velocity Profile (k-ε Model) Compared with Naudascher's Data

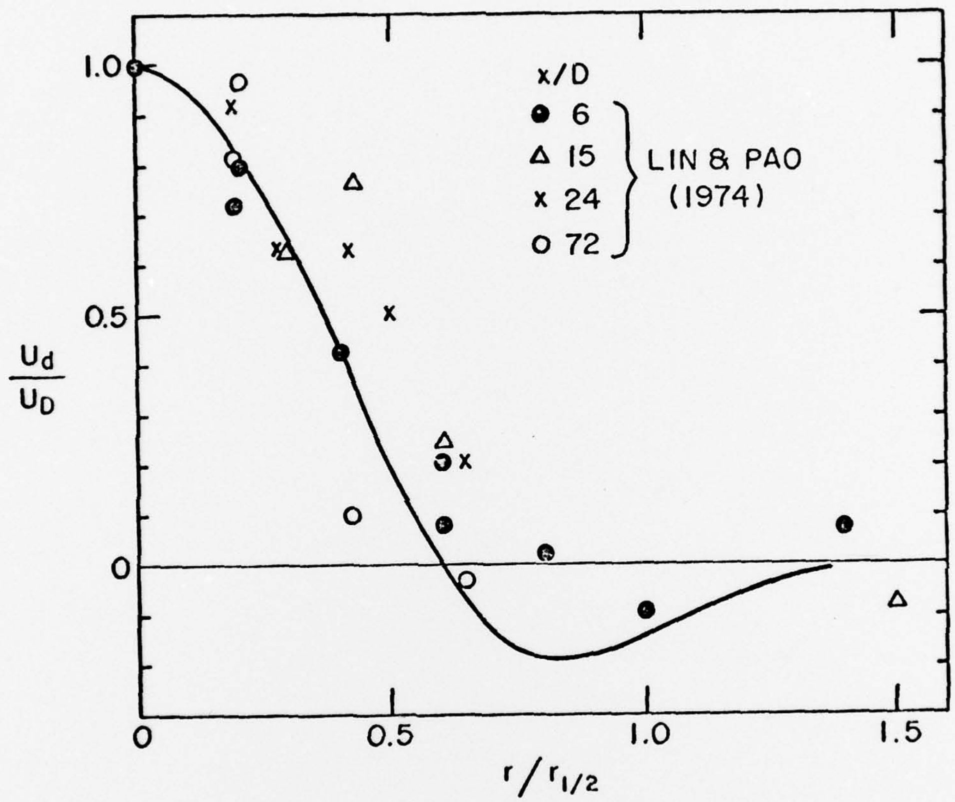

Figure 13a. Self-Similar Velocity Profile (k-ε Model) Compared with Lin & Pao's Data

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